

Scema Horizon
17/ENG03/024
Civil Engineering

ENG 381 Assignment

$$d^2x + 5 \frac{dx}{dt} + 6x = \cos t$$

C.F. $m^2 + 5m + 6 = 0$

$-b \pm \sqrt{b^2 - 4ac}$	$-\frac{5}{2} - \frac{1}{2}$	$-\frac{5}{2} + \frac{1}{2}$
$\frac{2a}{2a}$	$\frac{-5-1}{2}$	$\frac{-5+1}{2}$
$-\frac{5 \pm \sqrt{25 - 4(1)(6)}}{2(1)}$	$-\frac{6}{2}$	$-\frac{5+1}{2}$
$-\frac{5 \pm \sqrt{25 - 24}}{2}$	$-\frac{3}{2}$	$-\frac{4}{2} = -2$
$-\frac{5 \pm \sqrt{1}}{2}$		

$m = -3$ or -2

C.P. $Ae^{-3t} + Be^{-2t}$

Particular

$y = \cos t$

guess an ans $= C \cos t + D \sin t$

$\frac{dy}{dt} = -C \sin t + D \cos t$

$\frac{d^2y}{dt^2} = -C \cos t - D \sin t$

$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$

$(-C \cos t + 5D \cos t + 6C \cos t) + (-D \sin t - 5C \sin t + 6D \sin t) = \cos t$

$5C \cos t + 5D \cos t = \cos t$ ----- eqn(1)

$-5C \sin t + 1D \sin t = 0$ ----- eqn(2)

Solving Simultaneously

$5C + 5D = 1$

$-5C + 1D = 0$

$100 = 1$
 $0 = 10$

$5C + 5D = 1$

$(C = \frac{1}{2}, D = \frac{1}{2})$

$C = \frac{1}{2}, D = \frac{1}{2}$

$$\begin{aligned}
 \text{P.I. } x &= \frac{1}{2} \cos t + \frac{1}{2} \sin t \\
 x &= A e^{2t} + B e^{-2t} + \frac{1}{2} (\sin t + \cos t) \\
 \text{where } t=0; x=0.1 \neq \frac{1}{2} \sin 0 &= 0 \\
 0.1 &= A(1) + B(1) + \frac{1}{2}(1+1) \\
 0.1 &= A+B+1 \\
 A+B &= 0.1 - 2 = -1.9 \quad \text{--- eqn (3)}
 \end{aligned}$$

IC

$$\begin{aligned}
 x &= k \sin(\omega t + \phi) \\
 \text{where } x=0.1 \text{ at } t=0 \text{ and } \frac{dx}{dt} &= 0 \\
 \frac{dx}{dt} &= k \cos(\omega t + \phi) \\
 k \cos(\phi) &= 0 \\
 k \cos(\phi) &= 0 \\
 0.1 &= k \sin(\phi) \quad \text{--- eqn (2)} \\
 k \sin(\phi) &= 0.1
 \end{aligned}$$

$$\begin{aligned}
 \cos(\phi) &= 0 \Rightarrow \phi = \cos^{-1}(0) \\
 \phi &= 90^\circ
 \end{aligned}$$

Substitute ϕ into equation (1)

$$\begin{aligned}
 0.1 &= k \sin(90) \\
 k &= \frac{0.1}{\sin 90} \\
 k &= 0.1 \\
 x &= 0.1 (\sin(\omega t + 90))
 \end{aligned}$$

Command Window

```

Close all;
Clear all;
clc;
Sym b, x;
b = [0:0.1:15];
x = 0.1 * (sin(b + 90));
plot(b, x)

```