Assignment

Name: [illegible]

Matric: [illegible]

Dept: [illegible]

Q: $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxiliary equation => $k^2 + 5k + 6 = 0$

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+3)(k+2) = 0$

$\therefore k+3 = 0$ or $k+2 = 0$

$k = -3$ $k = -2$

Complementary function $= x_c = Ae^{k_1 x} + Be^{k_2 x}$

$y = Ae^{-3x} + Be^{-2x}$

P.I: $x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\Rightarrow -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$\Rightarrow -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

Comparing coefficient

Coefficient of cos: $-C + 5D + 6C = 1$ ... (1)

Coefficient of sin t: $-D - 5C + 6D = 0$ ... (2)

from (1)

$5C + 5D = 1$ ... (3)

from (2)

$5D - 5C = 0$ ... (4)

from (3) $5D = 1 - 5C$ ... (5)

substitute (5) into (4)

$(1 - 5C) - 5C = 0$

$1 - 5C - 5C = 0$

From ①: $5C + 5D = 1$ --- ③

" ②, $5D - 5C = 0$ --- ④

" ③; $5D = 1 - 5C$ --- ⑤

Subst. ⑤ into ④

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 = 0.1$

$\therefore 5D = 1 - 5C$

Subst. value of C in ⑤

$5D = 1 - 5(0.1)$

$5D = 0.5$

$D = 0.5/5 = 0.1$

$\therefore D = 0.1$

Particular Solution $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ --- ⑥

$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

From ⑥ $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

and $B = -A$

$B = -0.1$

General Solution $= 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$ //

2) - Command window
- clear
- clc
- closeall
- Symvar k t
- K = 0.1*exp(-3*t) - 0.1*exp(-2*t) + 0.1*cos(t) + 0.1*sin(t)
- t = 0, 0.01 : 15
- Kn = Subs(K)
- Plot (t, Kn)
- Xlabel('time')
- grid on
- grid minor
- axis tight

3) - $0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$ ⇒ steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos\alpha + K\sin\alpha\cos t$

Comparing coefficients

$\cos t : 0.1 = K\sin\alpha$

$\sin t : 0.1 = K\cos\alpha$

Square $K\sin\alpha$ and $K\cos\alpha$ and equate it to the addition

$K^2\sin^2\alpha + K^2\cos^2\alpha = 0.1 + 0.1$

$K^2(\sin^2\alpha + \cos^2\alpha) = 0.2$, Since $(\sin^2\alpha + \cos^2\alpha = 1)$

$\therefore K^2 = 0.2$

$K^2 = \frac{2}{100}$

$K = \frac{\sqrt{2}}{10}$

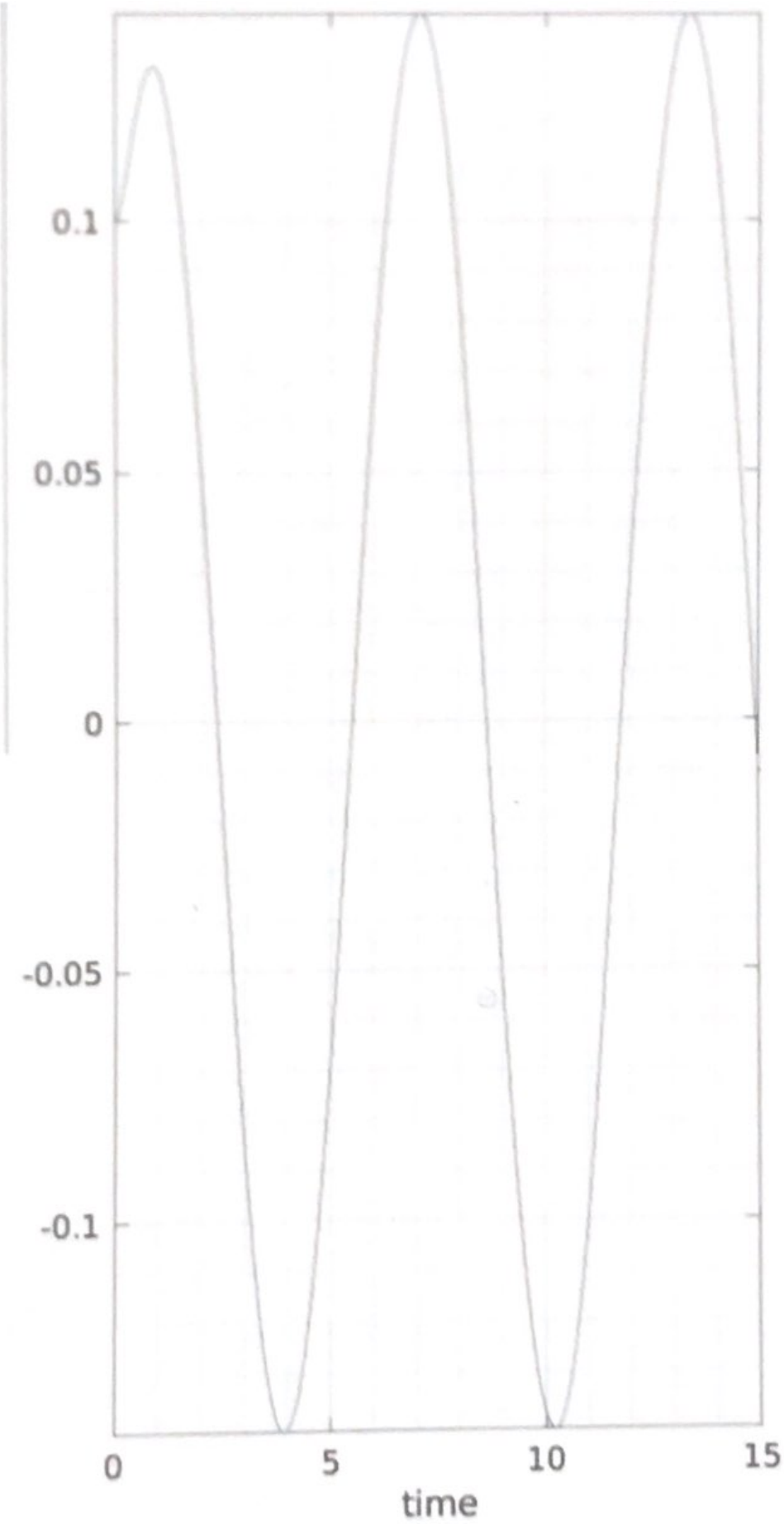
$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1}$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$ or $\pi/4$

$\therefore$ K steady state: $K_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$


0.1
0.05
0
-0.05
-0.1
0
5
10
15
time