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17/Engo1/1025
Chemical Engineering

Assignment 1

1) The dynamic model of a body in motion performing damped forced vibrations is as in Equation 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$

a) Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having $x$ as a function of $t$

b) With the aid of a matlab mfile program, plot the relationship between $x$ and $t$ for $0 \le t \le 15$ time unit using a step size of 0.01 unit and

c) Write the steady-state solution of the model in form of $x = K\sin(t+a)$

Solution

$$C.F = m^2 + 5m + 6 = 0$$
$$(m+3)(m+2) = 0$$
$$m = -3 \text{ and } m = -2$$
$$x = Ae^{-3t} + Be^{-2t}$$
$$f(t) = \cos t$$
$$x = C\cos t + D\sin t$$
$$dx/dt = -C\sin t + D\cos t$$
$$d^2x/dx^2 = -C\cos t - D\sin t$$

Subs

$$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$
$$= \cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$$
$$= \cos t(5D + 5C) + \sin t(5D - 5C) = \cos t$$

$$5D + 5C = 1$$
$$+5D - 5C = 0$$
$$10D = 1$$
$$D = 1/10$$

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{1}{2} + 5C = 1$

$C = \frac{1}{10}$

G.S $= x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Subs $(x = 0.1)$ $t = 0$

$0.1 = Ae^{-3(0)} + Be^{-2(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

$0.1 = A + B + 0.1$

$A + B = 0 \quad ---(1)$

$dx/dt = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + 0.1$

$-0.1 = 3A - 2B$

$3A + 2B = 0.1 \quad --(2)$

$2A + 2B = 0$

$-3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

To find B

$A + B = 0$

$0.1 + B = 0$

$B = -0.1$

G.S $= \frac{1}{10}e^{-3t} - \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

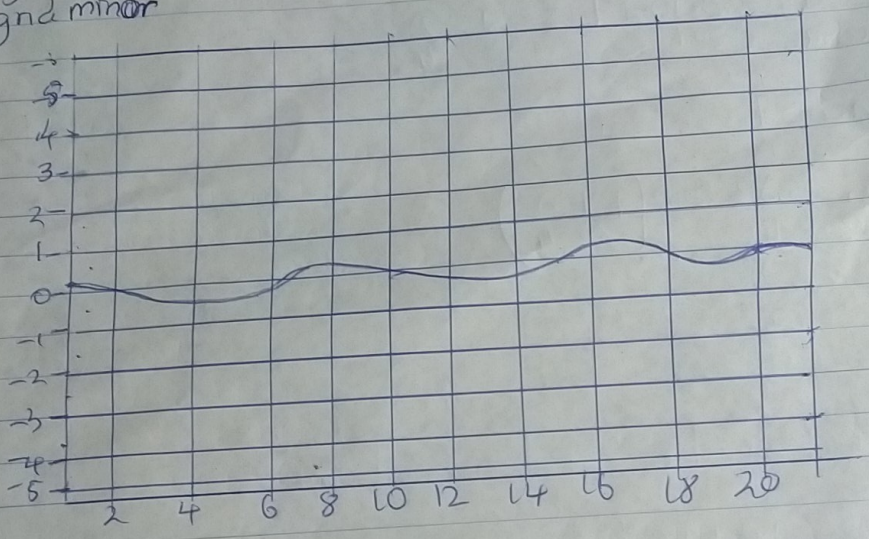
$= -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= \frac{1}{10}\left(e^{-2t} + e^{-3t} + \cos t + \sin t\right)$

B) MatLAB

Command window

```
Clear
Clc
Close all
Syms t
t = 0:0.01:15
x = 0.1*[exp(-2*t) + exp(-3*t) + cos(t) + sin(t)]
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor
```




C- steady state:

$x \; x_{t\to\infty} = 0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

recall $K\sin(t+\alpha) = K\sin t\cos\alpha + K\cos t\sin\alpha$

$\therefore x_{t\to\infty} = 0.1\cos t + 0.1\sin t = K\sin t\cos\alpha + K\cos t\sin\alpha$

Taking Coefficient of Cos and Sin

$0.1 = k\sin\alpha$

$0.1 = k\cos\alpha$

Square both sides

$k^2\sin^2\alpha + k^2\cos^2\alpha = \frac{1}{100} + \frac{1}{100}$

$k^2(\sin^2\alpha + \cos^2\alpha) = \frac{1}{50}$

$k^2 = \frac{1}{50}$ $\quad k = \frac{\sqrt{2}}{10}$

$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1} = 1 \therefore \alpha = \frac{\pi}{4}$

$\tan\alpha = 1$

$x_{ss} = \frac{\sqrt{2}}{10}\sin\left(t + \frac{\pi}{4}\right)$