

Let Auxiliary form

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) = 0$$

$$m+2 = 0 \quad \text{or} \quad m+3 = 0$$

$$x = Ae^{-2t} + Be^{-3t}$$

Particular Integral = $\cos t$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

When $t = 0$, $x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\sin(0) + \cos(0))$$

$$0.1 = A + B + 0 + \frac{1}{10}$$

$$A + B = 0 \dots \textcircled{1}$$

When $t = 0$, $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1 (\cos t - \sin t)$$

$$0 = 2A - 3B + 0.1$$

$$-0.1 = -2A - 3B \dots \textcircled{2}$$

Recall that $A + B = 0 \dots \textcircled{3}$

$$-0.1 = -2(-B) - 3B$$

$$x = 0.1 \cos t + 0.1 \sin t$$

$t = \infty$ Steady state

$$0.1 \cos t + 0.1 \sin t = K \sin(t + a)$$

$$K \sin(t) = K \sin t \cos a + K \cos t \sin a$$

Coefficient of $\cos t = K \sin a$

Coefficient of $\sin t = K \cos a$

Square both sides

$$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 a + \cos^2 a) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.1414$$

