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1a) C.F = $m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3$ and $m = -2$

$x = Ae^{-3t} + Be^{-2t}$

$f(t) = \cos t$

$x = C \cos t + D \sin t$

$\frac{dx}{dt} = -C \sin t + D \cos t$

$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$

subs

$= -C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$

$= \cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$

$= \cos t (5D + 5C) + \sin t (5D - 5C) = \cos t$

$5D + 5C = 1$

$+ 5D - 5C = 0$

$10D = 1$

$D = \frac{1}{10}$

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{1}{2} + 5C = 1$

$C = \frac{1}{10}$

G.S = $x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$

subs ($x = 0.1$) $t = 0$

$0.1 = Ae^{-3(0)} + Be^{-2(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$

$0.1 = A + B + 0.1$

$A + B = 0 \quad \text{--- (1)}$

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$

$0 = -3A - 2B + 0.1$

$-0.1 = 3A - 2B$

$3A + 2B = 0.1 \quad \text{--- (2)}$

$$\begin{aligned} 2A + 2B &= 0 \\ -3A + 2B &= 0.1 \\ \hline -A &= -0.1 \\ A &= 0.1 \end{aligned}$$

To find B

$$\begin{aligned} A + B &= 0 \\ 0.1 + B &= 0 \\ B &= -0.1 \end{aligned}$$

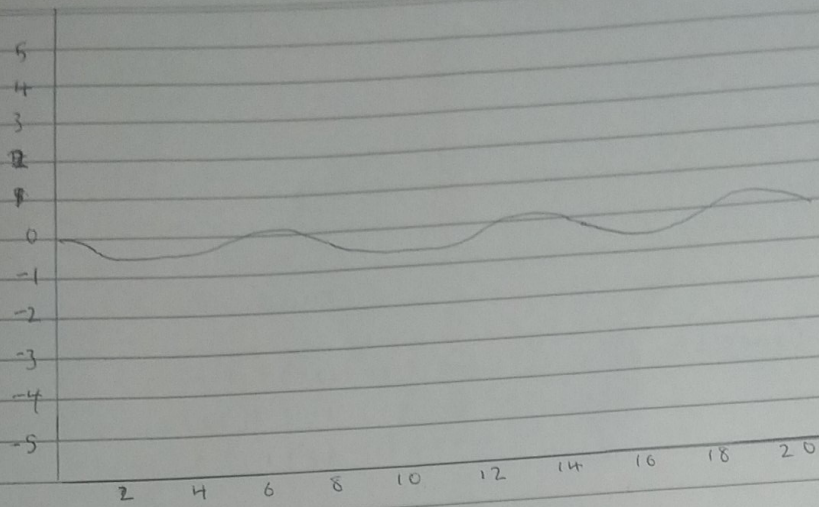
$$\begin{aligned} G(s) &= \frac{1}{10} e^{-3t} - \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t \\ &= \frac{-1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t \\ &= \frac{1}{10} (-e^{-2t} + e^{-3t} + \cos t + \sin t) \end{aligned}$$

b) Matlab

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command window
clear
clc
close all
syms t
t = 0:0.01:15
x = 0.1 [-exp(-2*t) + exp(-3*t) + cos(t) + sin(t)]
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor

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c) steady state

$$x \ll \infty \Leftrightarrow \infty = 0.1 \cos t + 0.1 \sin t = k \sin(t+\alpha)$$

$$\text{Recall } k \sin(t+\alpha) = k \sin t \cos \alpha + k \cos t \sin \alpha$$

$$\therefore x_{t \rightarrow \infty} = 0.1 \cos t + 0.1 \sin t = k \sin t \cos \alpha + k \cos t \sin \alpha$$

Taking coefficient of \cos & \sin

$$0.1 = k \sin \alpha$$

$$0.1 = k \cos \alpha$$

Square both sides

$$k^2 \sin^2 \alpha + k^2 \cos^2 \alpha = \frac{1}{100} + \frac{1}{100}$$

$$k^2 (\sin^2 \alpha + \cos^2 \alpha) = \frac{1}{50}$$

$$k^2 = \frac{1}{50} \quad k = \frac{\sqrt{2}}{10}$$

$$\frac{k \sin \alpha}{k \cos \alpha} = \frac{0.1}{0.1} = 1 \quad \therefore \alpha = \frac{\pi}{4}$$

$$\tan \alpha = 1$$

$$x_{ss} = \frac{\sqrt{2}}{10} \sin \left(t + \frac{\pi}{4} \right)$$