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Elect/Elect

17/ENR104/035

1) The dynamic model of a body in motion performing damped forced vibration is as in the equation below

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0$ and $\frac{dx}{dt}=0$

a) Using the Auxiliary Equation Method, obtain the solution of the model in form of an expression having x as a function of t .

b) With the aid of a MATLAB mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time units using a step size of 0.1 units, and

c) write the steady-state solution of the model in form of $x = K \sin(\omega t + \alpha)$

Sols:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$\therefore m_1 = -3 \text{ and } m_2 = -2$$

$$\Rightarrow C.F = Ae^{-3x} + Be^{-2x}$$

$$P.I = F(x) = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$5D \cos t - C \cos t + 6C \cos t - 5C \sin t - D \sin t + 6D \sin t = \cos t$$

$$5D \cos t + 5C \cos t - 5C \sin t + 5D \sin t = \cos t$$

$$5D + 5C = 1 \quad (i)$$

$$5D - 5C = 0 \quad (ii)$$

Using Simultaneous Eqns

$$5D + 5C = 1 \quad - (i)$$

$$+ 5D - 5C = 0 \quad - (ii)$$

$$10D = 1$$

$$D = \frac{1}{10}$$

Substitute $D = \frac{1}{10}$ in Eqn (i)

$$5\left(\frac{1}{10}\right) + 5C = 1, \quad \frac{1}{2} + 5C = 1$$

$$5C = \frac{1}{2} - \frac{1}{2} + 5C = 1 - \frac{1}{2}$$

$$5C = \frac{1}{2}$$

$$\therefore C = \frac{1}{2} \times \frac{1}{5}, \quad C = \frac{1}{10}$$

$$\Rightarrow \text{P.I.} = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$G.S = C.F + P.I$$

$$= Ae^{-3x} + Be^{-2x} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$= Ae^{-3x} + Be^{-2x} + \frac{1}{10} (\cos t + \sin t)$$

when $t=0$, $x=0.1$ and $\frac{dx}{dt} = 0$

$$\therefore 0.1 = A(1) + B(1) + \frac{1}{10}(1)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 - 0.1$$

$$A + B = 0 \quad - (1)$$

$$\frac{dx}{dt} = -3Ae^{-3x} - 2Be^{-2x} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad - (2)$$

\therefore Using simultaneous eqns

$$A + B = 0 \quad - (1) \quad \times 2$$

$$3A + 2B = 0.1 \quad - (2) \quad \times 1$$

$$\Rightarrow \begin{array}{r} 2A + 2B = 0 \\ - 3A + 2B = 0.1 \end{array}$$

$$-A = -0.1$$

$$A = 0.1$$

Substituting $A = 0.1$ in eqn (1)

$$A + B = 0$$

$$0.1 + B = 0$$

$$B = -0.1$$

$$\therefore G.S = 0.1e^{-3t} - 0.1e^{-2t} + \frac{1}{10} (\cos t + \sin t)$$

$$0.1 (e^{-3t} - e^{-2t} + \sin t + \cos t)$$

B) Command W
Clear
Clc
Close all
Symmat
 $t=0:0.1$
 $x=0:1$
 $X_n = \text{subs}$
Plot (t,
Axis title
grid on
grid min

c) x

b) Command Window

clear

clc

close all

syms t

~~t = 0:0.01:15~~

t = 0:0.01:15

x = 0.1 * (exp(-2*t) - exp(-3*t)) + cos(t) + sin(t)

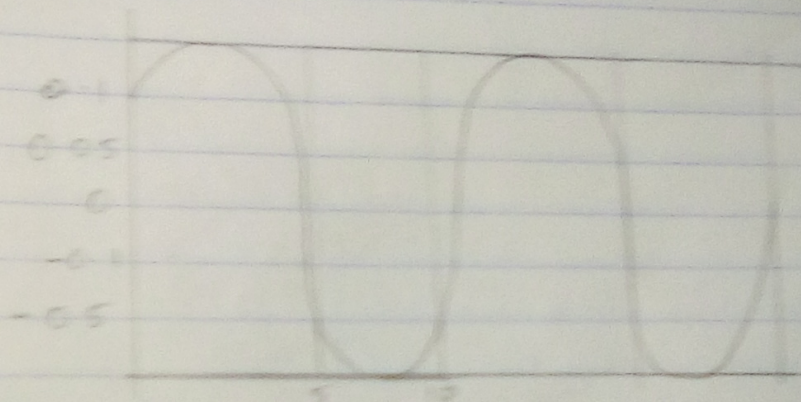
xn = subs(x)

Plot(t, xn)

axis tight

grid on

grid minor



c) $x = K \sin(\omega t + a)$

Knowing that $x = 0.1$ at $t = 0$ and $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = K \cos(\omega t + a)$$

$$0 = K \cos(\omega \cdot 0 + a)$$

$$\therefore K \cos(a) = 0$$

$$0.1 = K \sin(\omega \cdot 0 + a)$$

$$K \sin(a) = 0.1$$

$$\cos a = 0$$

$$\therefore a = \cos^{-1}(0) = 90^\circ$$

$$0.1 = K \sin(90^\circ)$$

$$\therefore K = 0.1$$

$$\therefore x = 0.1 [\sin(t+90)]$$

Command window

close all

clear

clc

Syms b, x

$$t = [0:0.01:15]$$

$$x = 0.1 * (\sin(t+90))$$

Plot (t,x)