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 COMPUTER ENGINEERING
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A. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

C.F $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) = 0$$

$$\therefore m = -2 \quad \text{and} \quad m = -3$$

C.F $x = Ae^{-2t} + Be^{-3t}$

P.I $\rightarrow x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C + 5D + 6C = 1 \quad \Rightarrow \quad 5D + 5C = 1 \quad \text{--- (i)}$$

$$-D - 5C + 6D = 0 \quad \Rightarrow \quad -5C + 5D = 0 \quad \text{--- (ii)}$$

$$\begin{aligned} 5C + 5D &= 1 \\ -5C + 5D &= 0 \end{aligned}$$

$$\therefore C = \frac{1}{10}$$

Substitute C into (i)

$$5\left(\frac{1}{10}\right) + 5D = 1$$

$$\frac{1}{2} + 5D = 1$$

$$5D = 1 - \frac{1}{2}$$

$$D = \frac{1}{10}$$

$$\frac{5D}{5} =$$

$$PI \quad x = \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{\cos(0)}{10} + \frac{\sin(0)}{10}$$

$$* 0.1 = A + B + \frac{1}{10}$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{\sin 0}{10} + \frac{\cos 0}{10}$$

$$* 0 = -2A - 3B - \frac{1}{10}$$

$$0.1 = A + B + \frac{1}{10} \quad \Rightarrow \quad A + B = 0.1 - \frac{1}{10}$$

$$0 = -2A - 3B - \frac{1}{10} \quad \Rightarrow \quad -2A - 3B = \frac{1}{10}$$

$$\begin{array}{r} -3A - 3B = 0 \\ -2A - 3B = 0.1 \\ \hline -A = -0.1 \end{array} \quad \therefore A = 0.1$$

$$0.1 + B = 0 \quad \therefore B = -0.1$$

\therefore General Solution

$$x = 0.1e^{-2t} - 0.1e^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

B. MATLAB CODE

command window

clear

clc

close all

syms x

syms t

t = 0:0.1:15

x = 0.1 * exp(-2*t) + 0.1 * exp(-3*t) + 0.1 * cos(t) + 0.1 * sin(t)

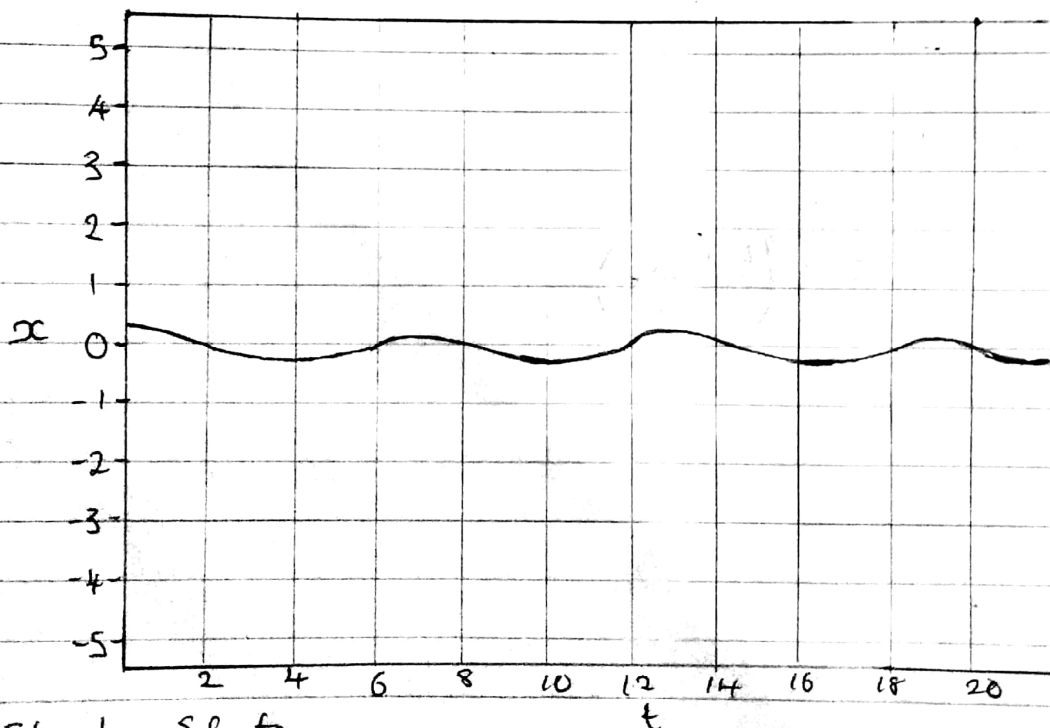
Plot (t, x)

x label ('x'), y label ('t'), title ('x-t graph')

grid on

axis equal

GRAPH



C. Steady-State

$$x_{ss} = x_{t \rightarrow \infty} = 0.1 \cos t + 0.1 \sin t = k \sin(t+a)$$

$$\text{Recall } k \sin(t+a) = k \sin t \cos a + k \cos t \sin a$$

$$\therefore x_{t \rightarrow \infty} = 0.1 \cos t + 0.1 \sin t = k \sin t \cos a + k \cos t \sin a$$

Taking the coefficients of \cos and \sin

$$0.1 = k \sin a$$

$$0.1 = k \cos a$$

Square both sides

$$k^2 \sin^2 a + k^2 \cos^2 a = \frac{1}{100} + \frac{1}{100}$$

$$k^2 (\sin^2 a + \cos^2 a) = \frac{1}{50}$$

$$k^2 = \frac{1}{50} \quad k = \frac{\sqrt{2}}{10}$$

$$\frac{k \sin a}{k \cos a} = \frac{0.1}{0.1} = 1 \quad \therefore a = \frac{\pi}{4}$$

$$\tan a = 1$$

$$x_{ss} = \frac{\sqrt{2}}{10} \sin \left(t + \frac{\pi}{4} \right)$$