i) The dynamic model of a body in motion performing damped forced vibration is as in the equation below

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

ii) Using the Auxiliary Equation Method, obtain the solution of the model in form of an expression having x as a function of t.

a) With the aid of a MATLAB m-file program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.1 unit, and

b) write the steady-state solution of the model in form of $x = K\sin(\omega t)$

Sol:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$\therefore m_1 = -3$ and $m_2 = -2$

$$\Rightarrow C.F = Ae^{-3x} + Be^{-2x}$$

$$P.I = F(x) = \cos t$$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6[C\cos t + D\sin t] = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$5D\cos t - C\cos t + 6C\cos t - 5C\sin t - D\sin t + 6D\sin t = \cos t$$

$$6D\cos t + 5C\cos t - 5C\sin t + 5D\sin t = \cos t$$

$$5D + 5C = 1 \quad \text{--- (i)}$$

$$5D - 5C = 0 \quad \text{--- (ii)}$$

Using Simultaneous eqn

$5D + 5C = 1$ — (i)

$5D - 5C = 0$ — (ii)

$10D = 1$

$D = \frac{1}{10}$

Substitute $D = \frac{1}{10}$ in Eqn (i)

$5(\frac{1}{10}) + 5C = 1$, $\frac{1}{2} + 5C = 1$

$5C = 1 - \frac{1}{2}$

$5C = \frac{1}{2}$

$\therefore C = \frac{1}{2} \times \frac{1}{5}$, $C = \frac{1}{10}$,,

$\Rightarrow$ P.I : $\frac{1}{10}\cos t + \frac{1}{10}\sin t$

G.S = C.F + P.I

$= Ae^{-3x} + Be^{-2x} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= Ae^{-3x} + Be^{-2x} + \frac{1}{10}(\cos t + \sin t)$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$\therefore 0.1 = A(1) + B(1) + \frac{1}{10}(1)$

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0$ — (i)

$\frac{dx}{dt} = -3Ae^{-3x} - 2Be^{-2x} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + \frac{1}{10}$

$3A + 2B = \frac{1}{10}$ — (2)

$\therefore$ Using simultaneous eqn

$A + B = 0$ — (i) $\times 2$

$3A + 2B = 0.1$ — (2) $\times 1$

$\Rightarrow$ $2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$,,

Substitute $A = 0.1$ in eqn (i)

$A + B = 0$

$0.1 + B = 0$

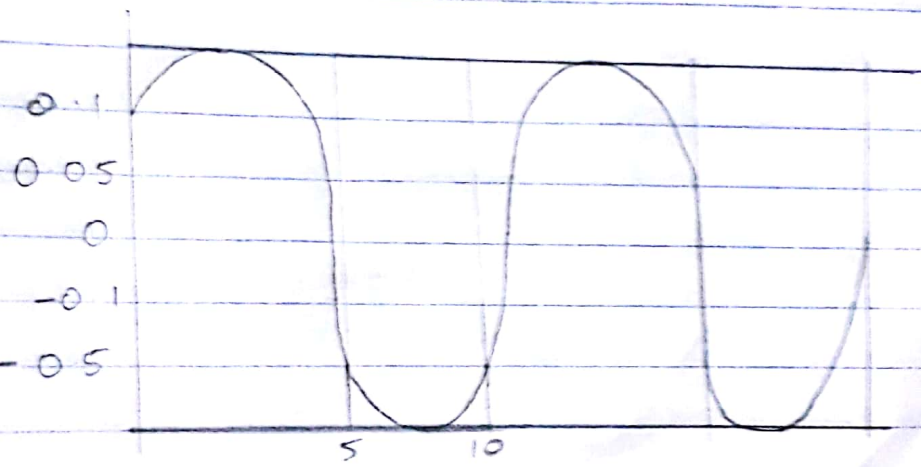
$B = -0.1$

$\therefore$ G.S = $0.1e^{-3t} - 0.1e^{-2t} + \frac{1}{10}(\sin t + \cos t)$

$0.1(e^{-3t} - e^{-2t} + \sin t + \cos t)$

b) Command Window

```
Clear
Clc
Close all
Syms t
t = 0:0.01:15
x = 0.1*(exp(-2*t) - exp(-3*t) + cos(t) + sin(t))
xn = subs(x)
Plot(t, xn)
axis tight
grid on
grid minor
```



c) $x = K\sin(t+a)$

Knowing that $x = 0.1$, at $t = 0$ and $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = K\cos(t+a)$$

$$0 = K\cos(0+a)$$

$$\therefore K\cos(a) = 0$$

$$0.1 = K\sin(0+a)$$

$$K\sin(a) = 0.1$$

$$\cos a = 0$$

$$\therefore a = \cos^{-1}(0)$$

$$= 90°$$

$$0.1 = K\sin(90°)$$

$$\therefore K = 0.1$$

$\therefore x = 0.1[\sin(t+90)]$

Command window

```
Close all
clear
clc
syms t, x
    t = [0:0.01.15]
    x = 0.1 * (sin[t+90])
Plot (t, x)
```