

OGUNLIFE ADEDAO ALMARTIN

CIVIL ENGINEERING

17/03/03

$$A \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Cx = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m_1 = -3 \quad m_2 = -2$$

$$y_{CF} = Ae^{-3t} + Be^{-2t}$$

$$F(x) = \cos t \quad y_{PI} = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t \quad \frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t + 0 \sin t$$

Comparing coefficients:

$$-C \cos t + 5D \cos t + 6C \cos t = \cos t$$

$$-D \sin t - 5C \sin t + 6D \sin t = 0 \sin t$$

$$-D - 5C + 6D = 0$$

$$-5C + 5D = 0$$

$$-5C = -5D$$

$$C = D$$

$$\cos t: -C \cos t + 5D \cos t + 6C \cos t = \cos t$$

$$-C + 5D + 6C = 1$$

$$5D + 5C = 1$$

$$5D = 1 - 5C$$

$$D = \frac{1 - 5C}{5}$$

$$5D = 1 - 5C$$

$$5D = 1 - 5D$$

$$10D = 1$$

$$D = 0.1$$

$$D = 0.1$$

$$C = D$$

$$C = 0.1$$

$$y_{PI} = 0.1 \cos t + 0.1 \sin t$$

$$y_{GS} = y_{CF} + y_{PI}$$

$$y_{GS} = Ae^{-3t} + Be^{-2t} + 0.1 [\cos t + \sin t]$$

~~$$y_{GS} = Ae^{-3t} + Be^{-2t} + 0.1 \cos t$$~~

$$y_{GS} = Ae^{-3t} + Be^{-2t} + 0.1 [\cos t + \sin t]$$

when $t=0$ $x=0.1$ $dx/dt=0$

$$x = Ae^{-3t} + Be^{-2t} + 0.1[\cos t + \sin t]$$

$$0.1 = Ae^{-3(0)} + Be^{-2(0)} + 0.1[\cos(0) + \sin(0)]$$

$$0.1 = A + B + 0.1$$

$$0.1 - 0.1 = A + B$$

$$A = -B$$

$$\frac{dx}{dt} = -3Ae^{-3t} + (-2Be^{-2t}) + 0.1[-\sin t + \cos t]$$

$$= -3(-B)e^{-3(0)} - 2Be^{-2(0)} + 0.1[-\sin(0) + \cos(0)]$$

$$= 3B - 2B + 0.1$$

$$= B + 0.1$$

$$B = -0.1$$

$$A = -B$$

$$A = -(-0.1)$$

$$A = 0.1$$

$$\therefore x_{a.s} = 0.1e^{-3t} + (-0.1e^{-2t}) + 0.1[\cos t + \sin t]$$

B) command window

clear

clc

close all

syms x, t

$$x = 0.1 * \exp(-3 * t) - 0.1 * \exp(-2 * t) + 0.1 * \cos(t) + 0.1 * \sin(t)$$

$$t = 0 : 0.01 : 15$$

$$Px = \text{subs}(t, x)$$

plot(t, Px)

xlabel('time')

grid on

grid minor

axis tight

$$c) 0.1 \cos t + 0.1 \sin t = k \sin(t + \theta)$$

$$0.1 \cos t + 0.1 \sin t = k \sin t \cos \theta + k \cos t \sin \theta$$

Comparing coefficients

$$\cos t : 0.1 = k \cos \theta$$

$$\sin t : 0.1 = k \sin \theta$$

Squaring $k \sin \theta$ and $k \cos \theta$ and equate it to the addition

$$k^2 \sin^2 \theta + k^2 \cos^2 \theta = 0.1 + 0.1$$

$$k^2 (\sin^2 \theta + \cos^2 \theta) = 0.2$$

$$k^2 = 0.2$$

$$k = \frac{\sqrt{2}}{10}$$

$$k \sin \theta = 0.1$$

$$k \cos \theta = 0.1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ \text{ or } \frac{\pi}{4}$$

k steady state

$$k_{ss} = \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right)$$