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Assignment 1

1. The dynamic model of a body in motion performing damped forced vibrations is an eqn(i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that when $t=0, x=0$ & $dx/dt = 0$

2. Using the auxiliary equation method, obtain the solution of the model in form of an expression having x as a function of t,

3. With the aid of a MATLAB m-file program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

4. Write the steady state solution of the model in form of

$$x = 16\sin(t+\alpha)$$

Solution

$$C.F = m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3, \quad m = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

$$f(t) = \cos t$$

$$x = C\cos t + \delta\sin t$$

$$\frac{dx}{dt} = -C\sin t + \delta\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - \delta\sin t$$

Also

$$= -C\cos t - \delta\sin t + 5(-C\sin t + \delta\cos t) + 6(C\cos t +$$

$D(\sin t) = \cos t$

$= \cos t\,(-C + 5D) + 6C) + \sin t\,(-D - 5C + 5D) = \cos t$

$= \cos t\,(5D + 5C) + \sin t\,(-5C + 5D) = \cos t$

$5D + 5C = 1$

$5D - 5C = 0$

$10D = 1$

$D = 1/10$

to find C

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{1}{2} + 5C = 1$

$C = \frac{1}{10}$

G.S $= x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Subs $(x = 0.1)t = 0$

$0.1 = Ae^{-0} + Be^{-0} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$0.1 = A + B + 0.1$

$A + B = 0$ -- eqn(1)

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + \frac{1}{10}$

$3A + 2B = \frac{1}{10}$ -- eqn(11)

$A + B = 0$ -- (2)

$3A + 2B = 0.1$ -- (3)

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

to find B

$0.1 + B = 0$

$B = -\frac{1}{10}$ or -0.1

$$G.S = \frac{-1}{10}e^{-3t} + \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$G.S = \frac{1}{10}e^{-2t} - \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$\frac{1}{10}\left(e^{-2t} - e^{-3t} + \cos t + \sin t\right)$$

iii) MATLAB mfile

```
Commandwindow
clear
clc
Close all
Syms t
    t = 0:0.01:15
    x = 0.1*[exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]
    xn = subs(x)
        plot(t, xn)
        axis tight
        grid on
        grid minor
```