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Mechanical Engineering

The diameter model of a body in motion performing damped forces vibration is a sin equation

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that $t=0$, $x=0.1$ and $dx/dt = 0$

a) Using the auxiliary method, obtain the solution of the model Inform-ation of an expression having x as a function of t.

b) With the aid of mathlab mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time ~~with~~ in t using a step size of 0.01 limit end

c) Write the steady state equation of the model Inform of $x = U\sin(t-\alpha)$

Solution

① $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

for c-f wher $dx/dt = a$, $x=1$, $f(t)=0$

$a^2 + 5a + 6 = 0$

$[3, 2]$

$a^2 + 3a + 2a + 6 = 0$

$a(a+3) + 2(a+3) = 0$

$(a+2)(a+3)$

$a = -2$, $a = -3$, real and different

$f(t) = Ae^{-2t} + Be^{-3t}$

for P.I

$y = \cos t$

where $x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

$\therefore x = \cos t$, $dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

where $d^2x/dt^2 + 5dx/dt + 6x = \cos t$

$(-C\cos t + D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\cos t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\therefore \sin t(5D - 5C) = 0$

$5D - 5C = 0$

$5D = 5C$

$D = C$

$\cos t(-C + 5D + 6C) = \cos t$

$5C + 5D = 1$

where $C = D$

$5D + 5D = 1$

$10D = 1$

$\therefore D = 1/10 \quad C = 1/10$

$x = 1/10(\cos t + \sin t)$

$\therefore$ General solution

$x = C.F + P.F$

$x = Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t)$ //

when $t = 0$, $x = 0.1$

$1/10 = Ae^{-2(0)} + Be^{-3(0)} + 1/10(\cos(0) + \sin(0))$

$1/10 = A + B + 1/10$

$1/10 - 1/10 = A + B \quad \ldots\ldots \quad A + B = 0 \quad \ldots\ldots (1)$

when $dx/dt = 0$, $x = 0.1$

$x = Ae^{2t} + Be^{-3t} + 1/10 \cos t + 1/10 \sin t$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} - 1/10 \sin t + 1/10 \cos t$

where $dx/dt = 0$, $x = 0.1$, $t = 0$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - 1/10 \sin(0) + 1/10 \cos(0)$

$(0 = -2A - 3B + 1/10) = 0 = 2A + 3B - 1/10$

$10(2A + 3B) = (1/10)\,10$

$20A + 30B = 1$ ..... (ii)

$A + B = 0$ ... (i)

$A = -B$ ..... (iii)

put (iii) in (ii)

$20(-B) + 30B = 1$

$-20B + 30 = 1$

$10B = 1$

$B = 1/10 \quad A = -1/10$

$\therefore x = 1/10\,(e^{-3t} - e^{-2t} + \cos t + \sin t)$

B) Command window

```
clc
close all
syms t
x = 1/10 * (exp(-3*t) * (cos(t) + (sin(t)))
t = 0 : 0.01 : 15
xt = subs (x, t)
xtn = double (xt
plot (t, xtn)
ylabel ('y'
```

ylabel ('x')

grid on

grid minor

grid right

c. At steady state

$t \xrightarrow{x} 0$ = steady state $= 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+n)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$