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DEPT: ELECTRICAL/ELECTRONICS ENGINEERING

MATRIC NO: 17/ENG04/032

ASSIGNMENT 1

1. The dynamic model of a body in motion performing damped force vibration is an equation 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$.

a.) Using the auxillary equation method, obtain the solution of the model in form of an expression having x as the function of t.

b.) With the aid of a MATLAB mfile program plot the relationship between x and t for $0 \leq t \leq 15$ time and a using a step size of 0.01 unit and

c.) write the steady state solution of the model in form of $x = k\sin(t+a)$

Solution

a.) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxillary Eqn : $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ Or -2

Complementary function : $x = Ae^{-3t} + Be^{-2t}$

Particular Integral : Assume $x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting Integral : Asm the value of d^2x/dt^2 & dx/dt in the eqn

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6((C\cos t + D\sin t) = \cos t$

comparing coefficient

Coefficient of cost : $-C + 5D + 6C = 1$ ... - (1)

Coefficient of sin : $-D - 5C + 6D = 0$ ... - (2)

from eqn (1) $5C + 5D = 1$ ... (3)

from eqn (2) $5D + 5C = 0$ ... (4)

From eqn (1) $5D = 1 - 5C$ ... (5)

put eqn (5) in eqn (4)

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 ; 0.1$

$\therefore 5B = 1 - 5C$

put the value of C in eqn(5)

$5B = 1 - 5(0.1)$

$5B = 0.5$

$B = \frac{0.5}{5} = 0.1$

Particular solution : $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ---------- (6)

$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from eqn (6) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

and $B = -A$

$B = -0.1$

General solution : $0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

2. Command window

```
clear
clc
close all
syms xt
x = 0.1 * exp(-3*t) - 0.1 * exp(-2*t) + 0.1 * cos(t) + 0.1 * sin(t)
t = 0:0.01:15
kn = subs(x)
plot(t, kn)
klabel ['time']
grid on
grid minor
axis tight
```

c. $0.1\cos t + 0.1\sin t = k\sin(t+a)$ at steady flow

$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\sin a\cos t$

Comparing Coefficient

Coefficient of cost : $0.1 = k\sin a$

Coefficient of sint : $0.1 = k\cos a$

square ksina and kcosa and equate it to the addition

$k^2\sin^2 a + k^2\cos^2 a = 0.1 + 0.1$

$k^2(\sin^2 a + \cos^2 a) = 0.2 \quad [\sin^2 a + \cos^2 a = 1]$

$k^2 = 0.2$

$k^2 = \frac{2}{100}$

$k = \frac{\sqrt{2}}{10}$

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1}$

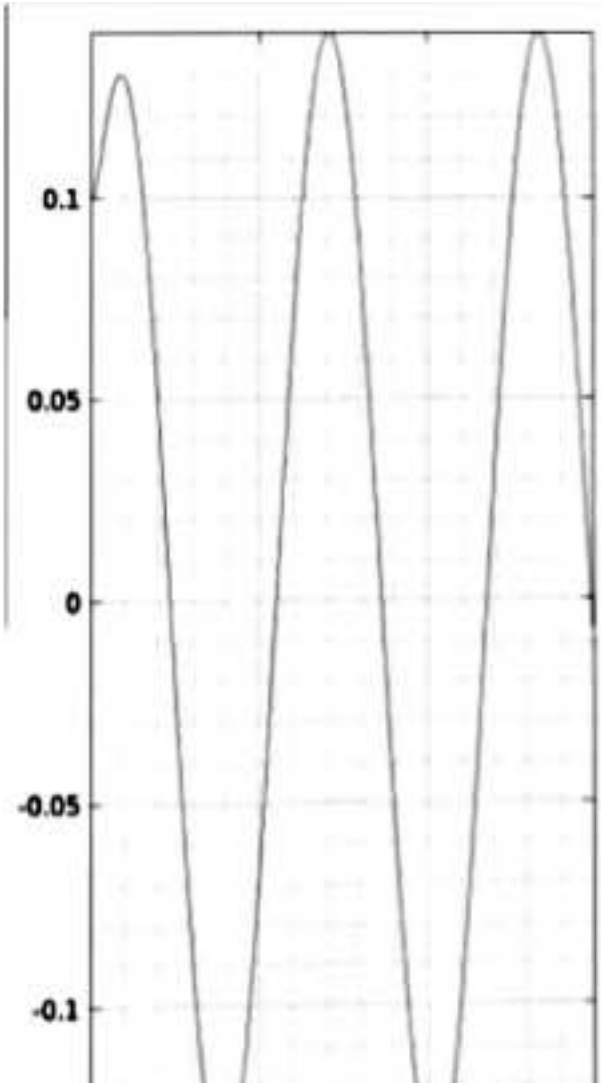
$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45^\circ$ or $\frac{\pi}{4}$

∴ k steady state;

$k_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$


0.1
0.05
0
-0.05
-0.1