

$$1) C.F_2 m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m_1 = -3, m_2 = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

$$f(t) = \cos t$$

$$x = c \cos t + D \sin t$$

$$\frac{dx}{dt} = -c \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -c \cos t - D \sin t$$

substitute

$$= -c \cos t - D \sin t + 5(-c \sin t + D \cos t) + 6(c \cos t + D \sin t) = \cos t$$

$$= \cos t (-c + 5D + 6c) + \sin t (-D - 5c + 6D) = \cos t$$

$$= \cos t (5D + 5c) + \sin t (-5c + 5D) = \cos t$$

$$5D + 5c = 1$$

$$5D - 5c = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

to find c

$$5\left(\frac{1}{10}\right) + 5c = 1$$

$$\frac{1}{2} + 5c = 1$$

$$c = \frac{1}{10}$$

$$G.S = x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

subst ($x=0.1$) $t=0$

$$0.1 = Ae^{-0} + Be^{-0} + \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \dots \text{eqn (1)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \dots \text{eqn (2)}$$

$$A + B = 0 \quad \dots \dots \dots$$

$$3A + 2B = 0.1 \times 1$$

$$2A + 2B = 0$$

$$3A + 2B = 0.1$$

$$-A = 0.1$$

$$A = 0.1$$

to find B

$$0.1 + B = 0$$

$$B = -\frac{1}{10} \text{ or } 0.1$$

$$G.S = \frac{-1}{10} e^{-3t} + \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$G.S = \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$= \frac{1}{10} (e^{-2t} - e^{-3t} + \cos t + \sin t)$$

iii) MATLAB file

command window

clear

clc

close all

syms t

t = 0:0.01:1.5

x = 0.1 * [exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]

xn = subs(x)

plot(t, xn)

axis tight

grid on

grid minor