

The dynamic model of a body in motion performing damped forced vibrations $D \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$

Given that when $t=0$, $x=0.1$ and $dx/dt = 0$

- Using the auxiliary eqn method, obtain the solution of the model in form of an expression having x as a function of t
- With the aid of a MATLAB mfile program, plot the relation between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit one
- Write the steady-state solution of the model in form of $x = k \sin(t + \alpha)$

Solution

$$D \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t \quad \text{--- (1)}$$

$$C.F = m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$\therefore m = -2 \quad \text{or} \quad m = -3$$

$$C.F = A e^{-2t} + B e^{-3t}$$

P.I

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substitute x , dx/dt , d^2x/dt^2 in eqn (1)

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t)$$

$$+ 6(C \cos t + D \sin t) = \cos t$$

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$\cos t (5C + 5D) + \sin t (5D - 5C) = \cos t$$

Equating coefficients

$$SD + SC = 1 \quad \text{--- 3}$$

$$SD - SC = 0 \quad \text{--- 4)}$$

from eqn 4)

$$SD = SC$$

$$D = C$$

substitute in eqn 3)

$$SD + SD = 1$$

$$10D = 1$$

$$D = 1/10$$

$$C = 1/10$$

$$P.2 \quad x = 1/10(\cos t + 1/10 \sin t) = 1/10(\cos t + \sin t)$$

The general solution is:

$$x = Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t) \quad \text{--- 5)}$$

when $t=0$, $x=0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10(\cos 0 + \sin 0)$$

$$0.1 = A + B + 1/10$$

$$1/10 = A + B + 1/10$$

$$A = -B \quad \text{--- 6)}$$

when $dx/dt = 0$

$$\frac{dx}{dt} = \frac{d}{dt}(Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t))$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$-2Ae^{-2t} - 3Be^{-3t} - 1/10 \sin t + 1/10 \cos t = 0$$

$$-2A \rightarrow t=0$$

$$-2A - 3B + 1/10 = 0$$

$$-(2A + 3B) = -0.1$$

$$2A + 3B = 0.1 \quad \text{--- 7)}$$

substitute 6) into 7

$$2(-B) + 3B = 0.1$$

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$$A = -0.1$$

Substituting values back in general solution

$$x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t)$$

5) MATLAB mfile program

Command window

clear

clc

close all

t = 0:0.01:15

x = 0.1 * (exp(-3*t) - exp(-2*t) + cos(t) + sin(t))

subplot(2,1,1)

plot(t, x)

xlabel('t')

ylabel('x')

grid on

grid minor

axis tight

