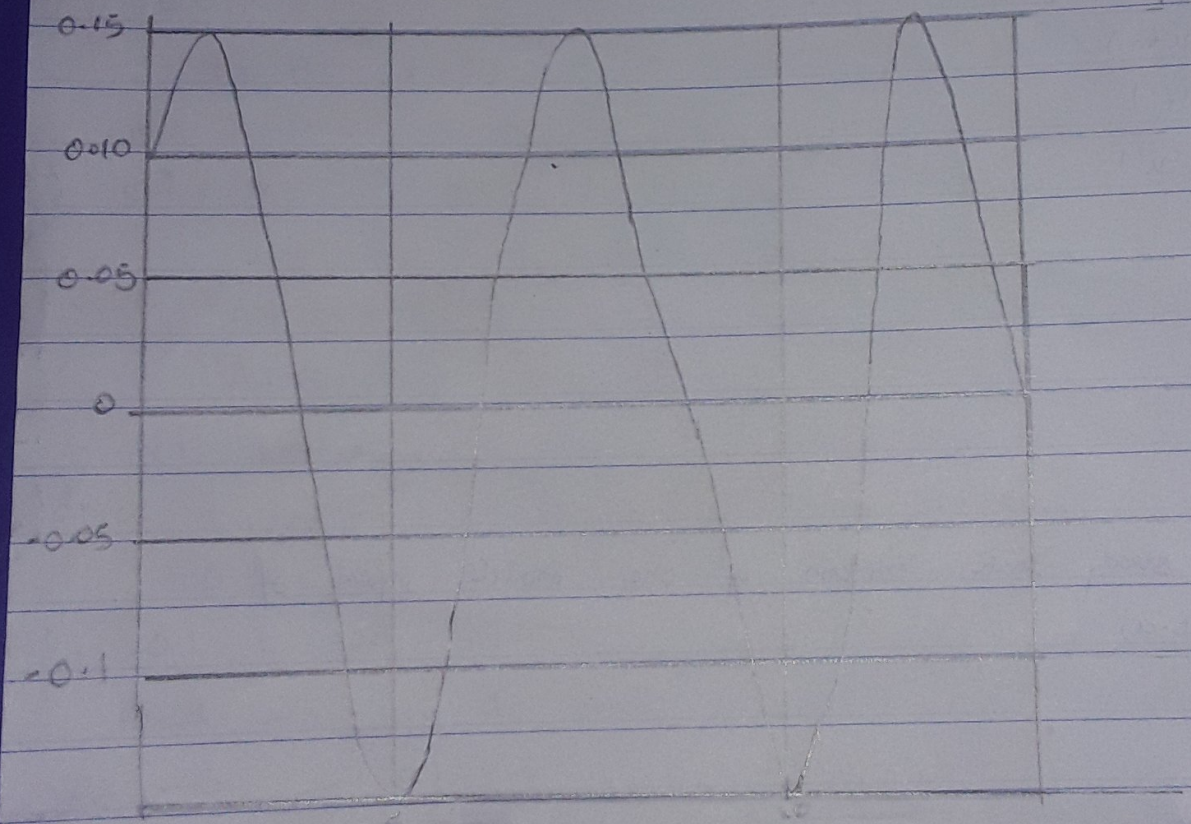
Steady state

$$= \frac{\sqrt{2}}{10} \sin(t + \pi/a)$$



- x = (1/15 * exp(-2*t)) - [1/10 * exp(-3*t)] + [1/10 (sin t) + cos t (x)]
- ~~t = 0:0.01:~~ · t = 0:0.01:15
- xt = subs (x, 0)
- x+n = double (x, t)
- Plot (t, x+n)
- x label ('t')
- y label ('x')
- grid on
- grid minor
- grid right

(5) Write one steady state solution of one model inform of

$x = K \sin(t-a)$

Solution;

- At steady state

$\lim_{t \to \infty} x = x_{\text{steady state}} = 0.1\cos t + 0.1 \sin t$

$0.1\cos t + 0.1 \sin t = K \sin(t+a)$

$K \sin(t+a) = K \sin t \cos a + K \cos t \sin a$

NB coefficient of $\cos t = K \sin a$

NB coefficient of $\sin t = K \cos a$

Squaring both sides;

$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02} = \sqrt{2}/10 \approx 0.144$

$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$

Recall that sin/cos = tan

$\tan a = 1$

$a = \tan^{-1}(1) \quad \therefore a = 45°$

$5c + 5(1/10) = 1$

$5c + 1/2 = 1$

$\therefore c = \frac{1}{2} \times \frac{1}{5}$

$c = 1/10$

$\therefore x = Ae^{-2t} + Be^{-3t} + 1/10\ [\sin t + \cos t]$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10\ [\sin(0) + \cos(0)]$

$0.1 = A + B + 1/10 + 0$

$A + B = 0.1 - 0.1$

$A + B = 0$ — (i)

when $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$\therefore -0.1 = -2A - 3B$

Recall with equ (i) that $A + B = 0$

$A = -B$ — (ii)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$\therefore B = 0.1$

Knowing that $A = -B$ as seen in equ (ii)

$\therefore A = -0.1$

$\Rightarrow x = -0.1e^{-2t} + 0.1e^{3t} + 1/10\ [\sin t + \cos t]$

(2) With the aid of matlab, m file program, plot the relationship between x and t for $0 \leq t \leq 15$ time in t using a step size of 0.01 unit end.

Solution

- Command window
- Clear
- clc
- close all
- syms t

ENG 318

ADAH EMMANUEL 17/ENG06/001 MECHANICAL ENGR.

① The dynamic model of a body in motion performing damped forced vibration is a sin equation;

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that; $t=0$, $x=0.1$ and $dx/dt = 0$

2) Using one auxiliary equation method, obtain one solution of the model information of an expression having x as a function of t.

Solution

1a) $$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t \quad \text{(a)}$$

In auxiliary form

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$m = -2, -3$$

$$\Rightarrow x = Ae^{-2t} + Be^{-3t}$$

$$P_I = \cos t$$

$$x = C\cos t + D\sin t$$

$$\therefore \frac{dx}{dt} = -C\sin t + D\cos t \quad \text{(c)}$$

Substitute (c) back into equ. (a)

$$-[C\cos t - D\sin t + 5C - C\sin t + D\cos t] + 6(C\cos t + D\sin t) = \cos t$$

$$-[C\cos t - D\sin t - 5C\sin t + 6\cos t + 6D\cos t + 6D\sin t] = \cos t$$

$$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\sin t - 5D\sin t + 5D\cos t$$

$$5C\cos t + 5D\cos t = \cos t$$

$$5C + 5D = 1 \quad \cdots \quad \text{(a)}$$

$$5D\sin t - 5C = 0$$

$$5D - 5C = 0 \quad \cdots \quad \text{(b)}$$

Simultaneously solving; $5C + 5D = 1$ $\therefore D = 1/10$

$$-5C + 5D = 0$$

$$10D = 1$$