YAHUBU HEEMMAT OHUNENE
17/ENG02/080
COMPUTER ENGINEERING
ENG 381

ASSIGNMENT

→ The dynamic model of a body in motion performing damped forced vibration is as in the equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that $x = 0.1$, $t = 0$ and $\frac{dx}{dt} = 0$

– Using the auxiliary equation method, the solution of the model in form of an expression having x as a function of t.

Solu

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$m^2 + 5m + 6 = 0$ Auxiliary equation

$m^2 + 3m + 2m + 6 = 0$

$(m+2)(m+3) = 0$

$m = -2$ or -3

Complimentary Function

$y = Ae^{-2t} + Be^{-3t}$

Particular Integral

$x = \cos t\, x$

$= C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Put

$= -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

Coefficients of cost.

$= -C\cos t + 6C\cos t + 5D\cos t = 1$

$= 5C + 5D = 1 \quad \cdots (1)$

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coefficents of sint

$-D\sin t + 6D\sin t - 5C\sin t = 0$

$5D - 5C = 0 \cdots (2)$

$5C + 5D = 1$

from equ (2)

$5D - 5C = 0$

$D = C \cdots (3)$

Put equ (3) in (1)

$5C + 5(C) = 1$

$10C = 1$

$C = 0.1$,

$D = 0.1$.

P.I

$x = 0.1\cos t + 0.1\sin t$

General solution

$x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$

When $t = 0$, $x = 0.1$

$0.1 = A + B + 0.1$

$A + B = 0$

When $t = 0$, $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1\sin t + 0.1\cos t$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$

from

$A + B = 0$

$B = -A$

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$-0.1 = -2A - 3(-A)$

$-0.1 = A$

$A = -0.1$

$B = 0.1$

Particular Function

$x = 0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$

→ Using MATLAB

Command window

```
clear
clc
close all
syms t
x = (1/10)*exp(-2*t) - (1/10)*exp(-3*t) + (1/10)*(sin(t) + cos(t))
t = 0:0.01:15
xt = subs(x, t)
xtn = double(xt)
plot(t, x, t, xtn)
xlabel('t')
ylabel('x')
grid on
grid minor
grid right
```

→ At steady state $K\sin(t+\alpha)$

$x_{ss} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t$

$= 0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

$x_{ss} = K\sin t\cos\alpha + K\cos t\sin\alpha$

$0.1 = K\sin\alpha$

$0.1 = K\cos\alpha$

$K^2\sin^2\alpha + K^2\cos^2\alpha = \frac{1}{100} + \frac{1}{100}$

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$$K^2(\sin^2\alpha + \cos^2\alpha) = 2/100$$

$$K^2 = \frac{2}{100}$$

$$K = \frac{\sqrt{2}}{10}$$

Divide both equations

$$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1} = 1$$

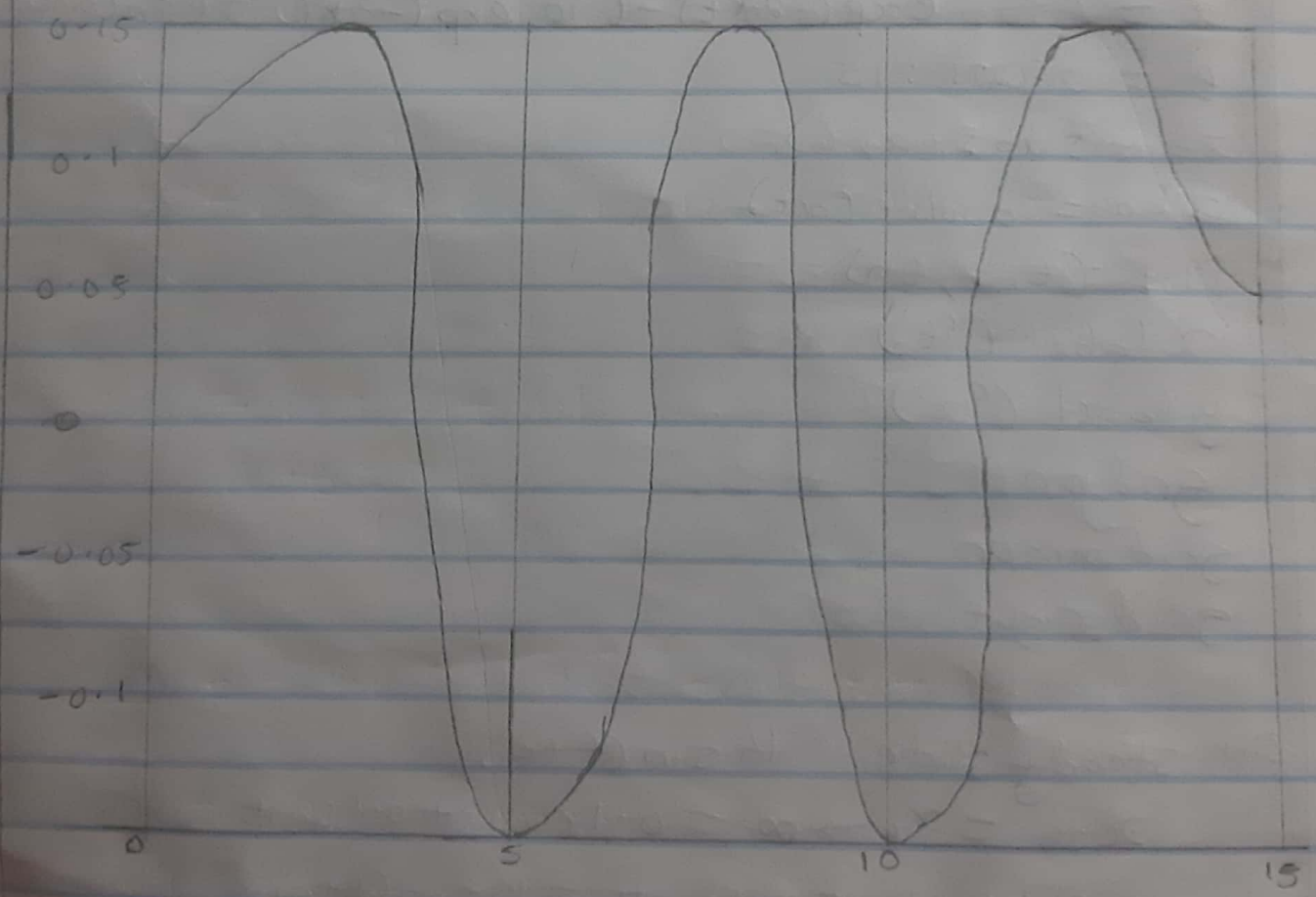
$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$\alpha = 45$ or $\pi/4$ radian

steady state

$$x_{ss} = \frac{\sqrt{2}}{10}\sin\left(t + \frac{\pi}{\alpha}\right)$$



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