

Maths ENG381 Assignment:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0 \quad (\text{C.F.})$$

Using completing the square method.

$$m + 5m + \left(\frac{-5}{2}\right)^2 = -6 + \frac{25}{4}$$

$$\left(m + \frac{5}{2}\right)^2 = \frac{1}{4}$$

$$m + \frac{5}{2} = \pm \frac{1}{2}$$

$$m = -3 \quad n = -2$$

$$x = \text{C.F.} = Ae^{-3t} + Be^{-2t}$$

Particular Integral.

$$x = C \cos t + D \sin t$$

$$\frac{dy}{dx} = -C \sin t + D \cos t$$

$$\frac{d^2y}{dx^2} = -C \cos t - D \sin t$$

$$C - (C \cos t - D \sin t) + 5(C - C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$5(C \cos t + 3D \cos t) = \cos t \dots \text{equ 1.}$$

$$C - (C \cos t + 5D \cos t + 6(C \cos t) + (-D \sin t - 5(C \sin t + 6D \sin t)) = \cos t - 5C$$

$$\sin t + 5D \sin t = 0 \dots \text{equ 2.}$$

Simultaneous equation.

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$C = -\frac{1}{2} \times \frac{1}{5}$$

$$C = -\frac{1}{10}$$

$$\text{P.I.} = x = -\frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\sin t - \cos t)$$

when $t=0$, $x=0.1$ & $\frac{dy}{dx} = 0$

$$0.1 = A(1) + B(1) + 1/10(1-1)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \dots \dots \text{equation 3.}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + 1/10(\cos t + \sin t)$$

$$0 = -3A - 2B + 1/10(1)$$

$$-0.1 = -3A - 2B$$

$$3A + 2B = 0.1 \dots \dots \text{equ 4}$$

Thus;

$$A + B = 0.2$$

$$A = 0.2 - B \text{ equ 5}$$

Sub-equation (5) into Equ 4

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$\therefore x = -0.3e^{-3t} + 0.1e^{-2t} + 1/10(\sin t - \cos t)$$

$$= 0.1e^{-2t} - 0.3e^{-3t} + 1/10(\sin t - \cos t)$$

B. Command Window

close all;

clear all;

clc

Sym x, t

t = [0:0.1:15];

x = 0.1 * exp(-2*t) - 0.3 * exp(-3*t) + 0.1 * (sin(t) - cos(t))

Plot (t, x)

1c. $x = k \sin(Ct + a)$

knowing $x = 0.1$ at $t = 0$ & $dx/dt = 0$

$$\frac{dx}{dt} = k \cos(t + a)$$

$$0 = k \cos(0 + a)$$

$$k \cos(a) = 0$$

$$0 = k \sin(0 + a) \dots \text{equ 1}$$

$$k \sin(a) = 0.1$$

$$\cos(a) = 0 \quad \Rightarrow \quad a = \cos^{-1}(0)$$

$$a = 90^\circ$$

Sub. a into equation 1

$$0.1 = k \sin(90)$$

$$k = 0.1 / \sin 90$$

$$k = 0.1$$

$$x = 0.1 (\sin(t + 90))$$

162. Command window

Close all;

Clear all;

Cle;

sym t, x

t = [0:0.1:15];

x = 0.1 (sin(t + 90));

Plot (t, x)