

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that $t=0, x=0.1, \frac{dx}{dt}=0$

$$0 = m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3, m = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

Particular cost

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Subs.

$$= -C \cos t - D \sin t + 5C \cos t + 5D \sin t + 6(C \cos t + D \sin t) = \cos t$$

$$= \cos t (-C + 5D + 6C) + \sin t (C - D - 5C + 6D) = \cos t$$

$$= \cos t (5C + 5D) + \sin t (C - 5C + 5D) = \cos t$$

$$5D + 5C = 1$$

$$5D - 5C = 0$$

$$D = 1/10; D = 1/10$$

find C

$$5C + 1/10 = 1$$

$$1/2 + 5C = 1$$

$$C = 1/10$$

$$G.S. > x = Ae^{-3t} + Be^{-2t} + 1/10 \cos t + 1/10 \sin t$$

Subs. $(x=0) t=0$

$$0 = Ae^{-0} + Be^{-0} + 1/10 \cos 0 + 1/10 \sin 0$$

$$0 = A + B + 0.1$$

$$A + B = 0 \quad \text{--- eqn 2}$$

$$\frac{dx}{dt} = -3xe^{-2t} - 2Be^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{--- eqn (1)}$$

$$4B = 0 \quad \times 2$$

$$3A + 2B = 0 \quad \times 1$$

$$2A + 2B = 0$$

$$3A + 2B = 0$$

$$-A = -0.1$$

$$A = 0.1$$

To find B

$$0.1 + B = 0$$

$$B = \frac{-0.1}{1} \text{ or } -0.1$$

$$0.5 = \frac{-1}{10} e^{-2t} + \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$0.5 = \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$= \frac{1}{10} (e^{-2t} - e^{-2t} + \cos t + \sin t)$$

= MATLAB mfile

Command window

clear

clc

close all

syms t

t = 0:0.01:15

X = 0.1 * [exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]

Xn = subs(X)

plot(t, Xn)

axis tight

grid on

grid minor