

Assignment 1

a. Define differential equation and give 2 examples.
 Differential equation is the relationship between an independent variable x , or dependent variable y and one or more derivative of y with respect to x .

Examples: 1) $x \frac{dy}{dx} - 3y = x^2$

2) $(x^2 + xy) \frac{dy^2}{dx^2} = xy - y^2 \frac{dy}{dx}$

b. i) $y = Ae^{-4x} + Be^{-6x}$

ii) Second order of the differential equation can be formed.

iii) This is due to the presence of 2 arbitrary constants A and B.

iv) $y = Ae^{-4x} + Be^{-6x}$ — (1)

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ — (2)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ — (3)

From eq (1) $Ae^{-4x} = y - Be^{-6x}$ — (4)

Substitute (4) on eq (2)

$\frac{dy}{dx} = -4(y - Be^{-6x}) - 6Be^{-6x}$

$\frac{dy}{dx} = -4y + 4Be^{-6x} - 6Be^{-6x}$

$\frac{dy}{dx} = -4y - 2Be^{-6x}$ — (A)

From eq (1) $Be^{-6x} = y - Ae^{-4x}$ — (5)

Substitute (5) in (A) $\frac{dy}{dx} = -4Ae^{-4x} - 6(y - Ae^{-4x})$

$\frac{dy}{dx} = -4Ae^{-4x} - 6y + 6Ae^{-4x}$

$$\frac{dy}{dx} = -6y + 2Ae^{-4x} \quad \text{--- (8)}$$

$$\text{from eqn (A): } 2Be^{-6x} = -4y - \frac{dy}{dx} \quad \text{--- (9)}$$

$$\text{from eqn (B): } 2Ae^{-4x} = \frac{dy}{dx} + 6y \quad \text{--- (10)}$$

Substitute (10) and (10) in eqn (9)

$$\frac{d^2y}{dx^2} = 8 \left(\frac{dy}{dx} + 6y \right) + 18 \left(-4y - \frac{dy}{dx} \right)$$

$$\frac{d^2y}{dx^2} = \frac{8dy}{dx} + 48y - 72y - 18 \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -10 \frac{dy}{dx} - 24y$$

$$\text{ANS: } \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0.$$