Akanmu-Fawaz Babajide
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Mechatronics Engineering

Answer

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x}\frac{d}{dx}(2x+1)$ (Product rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$
$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$ => proven.

1b) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem.

$w_1 = y''$
$w_2 = y'(2x+1)$
$w_3 = 2y$

de generate eqn = $w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $u' = y'''$ hence $u^n = y^{n+2}$
$v = 1$ $v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$
$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$ ..... (1)

w_2

$w_2 = y'(2x+1)$
$u = y'$ $u' = y''$ $u''' = y''''$, Hence $u^n = y^{n+1}$
$v = 2x+1$ $v' = 2$ $v'' = 0$
$w_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \ldots$

$$w_2 = y^{n+1}(2x+1) + n\,y^n(2) + \frac{n(n-1)}{2!}y^{n-2}(0) + \cdots$$

$$w_2 = y^{n+1}(2x+1) + 2ny^n + 0.$$

$$w_2 = y^{n+1}(2x+1) + 2ny^n. \quad \cdots \quad (2).$$

<u>w_3</u>

$u = y^n \qquad u' = y' \qquad$ Hence $u^n = y^n$

$v = 2. \qquad v' = 0$

$$w_3 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots$$

$$w_3 = y^n(2) + n y^{n-1}(0) + \cdots$$

$$w_3 = 2y^n. \quad \cdots \quad (3)$$

Putting back into the degenerate eqn.

$$w_1 - w_2 - w_3 = 0.$$

$$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y^n = 0.$$

$$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0.$$

$$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n.$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1).$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \implies \text{Proven.}$$

Question 2.

a.) $y = x^3 e^{4x}$.

Using Leibnitz theorem.

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$, $u^{iv} = 256e^{4x}$

$v = x^3$ $v' = 3x^2$ $v'' = 6x$ $v''' = 6$ $v^{iv} = 0$.

Hence $u^n = 4^n e^{4x}$.

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4 + \ldots$$

$$y^n = 4^n e^{4x}(x^3) + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x}(6x) + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x}(6)$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)^2.$$

$$y^n = 4^n x^3 e^{4x} + n 3x^2 4^{n-1} e^{4x} + 3x n(n-1) 4^{n-2} + n(n-1)(n-2) 4^{n-3} e^{4x} + 0$$

~~$y^n = 4^n x^3 e^{4x} +$~~

$$y^n = e^{4x} 4^{n-3} \left(x^3 4^3 + 3n x^2 4^2 + 3x n(n-1) 4 + n(n-1)(n-2)\right).$$

$$y^n = e^{4x} 4^{n-3} \left(64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2)\right) - (y^n.$$

$$y^5 = e^{4x} 4^{5-3} \left(64x^3 + 48x^2(5) + 12x\, 5(5-1) + 5(5-1)(5-2)\right)$$

$$y^5 = e^{4x} 4^2 \left(64x^3 + 240x^2 + 240x + 36\right).$$

$$y^5 = 16 e^{4x} \left(64x^3 + 240x^2 + 240x + 56\right)$$

2b). $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

$x^2y'' + xy' + y = 0$.

$w_1 + w_2 + w_3 = 0$.

$w_1 = x^2 y''$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^{v}$ Hence, $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$. $v'' = 2$. $v''' = 0$.

$w_1 = u^n(v) + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2}(v'') + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \cdots$

$w_1 = y^{n+2}(x^2) + n y^{n+1}(2x) + \frac{n(n-1)}{2} y^n(2) + 0$

$w_1 = x^2 y^{n+2} + 2xny^{n+1} + y^n n(n-1)$

$w_2 = xy'$

$u = y'$, $u' = y''$, $u'' = y'''$ Hence $u^n = y^{n+1}$.

$v = x$ $v' = 1$ $v'' = 0$.

$w_2 = u^n(v) + n u^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v''$

$w_2 = y^{n+1}(x) + n y^n(1) + 0$.

$w_2 = xy^{n+1} + ny^n$

$w_3 = y$.

$u = y$ $u' = y'$ Hence $u^n = y^n$.

$v = 1$ $v' = 0$

$w_3 = u^n(v) + n u^{n-1}(v')$

$= y(1) + 0 = y$.

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$