Ebosa Ofeoritse O.
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① If $y = e^{x^2+x}$,

$\cancel{k=1}$ $\cancel{u = e^{x^2+x}}$, $\cancel{v^{(x)} = 0}$, $\cancel{u^{(x)} = e^{x^2+x}}$

Differenciating using function of a function

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = (2x+1) \cdot e^u$

$y' = e^{(x^2+x)} \cdot (2x+1)$

differenciating using product rule

$y'' = \frac{d}{dx}[(2x+1) \times e^{x^2+x}] = \frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$

Let $u = 2x+1$, $v = e^{x^2+x}$

$\frac{du}{dx} = 2$

$v = e^{x^2+x}$ using function of a function

Let $z = x^2 + x$

$dz/dx = 2x+1$

$v = e^z$, $dv/dz = e^z$

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$

Substituting into the product formula

$y'' = v\frac{du}{dx} + u\frac{dv}{dx}$

$= e^{x^2+x} \cdot 2 + 2x+1 \cdot (2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + (2x+1)((2x+1)e^{x^2+x})$

But $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2y + 2x+1 \cdot y'$

rewriting, $y'' = y'(2x+1) + 2y \quad -(1)$

from eq (1), $y'(2x+1) + 2y - y'' = 0$

let $w_1 = y'(2x+1)$

$w_2 = 2y$

$w_3 = y''$

$w_1 = y'(2x+1)$

$v = 2x+1, \quad v^{(1)} = 2, \quad v^{(2)} = 0$

$u = y', \quad u^{(1)} = y^{(2)}, \quad u^{(2)} = y^{(3)}$

$u^n = y^{n+1}$

$w_1^n = u^n v^0 + n u^{n-1} v^1$

$w_1^n = y^{n+1} \cdot 2x+1 + n y^n \cdot 2$

$w_1^n = \cancel{y^{n+1}\, 2x+1} \quad y^{n+1}(2x+1) + 2n y^n$

$w_2 = 2y$

$u = y, \quad u' = y^{(1)} \qquad u^n = y^n$

$v = 2, \quad v^{(1)} = 0$

$w_2^n = u^n v^0$

$= y^n \cdot 2$

$= 2y^n$

$w_3 = y''$

$v = -1, \quad v^{(1)} = 0$

$u = y'', \quad u' = y^{(3)}, \qquad u^n = y^{n+2}$

$w_3^n = u^n v^0$

$= y^{n+2} \cdot -1$

$= -y^{n+2}$

$y^n = w_1 + w_2 + w_3$

$y^n = y^{n+1}(2x+1) + 2n y^n + 2y^n - y^{n+2} = 0$

Collecting like terms

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n = 0$

3 (1) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$

$v^{(0)} = x^3,\ v^{(1)} = 3x^2,\ v^{(2)} = 6x,\ v^{(3)} = 6,\ v^{(4)} = 0$

$u^{(0)} = e^{4x},\ u^{(1)} = 4e^{4x},\ u^{(2)} = 16e^{4x},\ u^{(3)} = 64e^{4x},\ u^{(4)} = 256e^{4x}$

$u^n = 4^n e^{4x}$

$y^n = u^n v^{(0)} + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)}$

$= 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} \cdot 6$

$y^n = [x^3 4^n e^{4x}] + [3x^2 n 4^{n-1} e^{4x}] + [4^{n-2} e^{4x} \cdot n(n-1) \cdot 3x] + [n(n-1)(n-2) 4^{n-3} e^{4x}]$

$y^n = 4^{n-3} e^{4x} [3x^2 \cdot n 4^2 + 4^3 x^3 + n(n-1) 4 \cdot 3 + n(n-1)(n-2)]$

$y^n = 4^{n-3} e^{4x} [64x^3 + 48n x^2 + 12n(n-1) + n(n-1)(n-2)]$

$y^5 = 4^{5-3} e^{4x} [64x^3 + 48(5)x^2 + 12(5)(5-1) + 5(5-1)(5-2)]$

$y^5 = 16e^{4x} [64x^3 + 240x^2 + 240 + 60]$

(2)(ii) $x^2 y'' + xy' + y = 0$

$y = w_1 + w_2 + w_3$

$w_1 = x^2 y'',\ w_2 = xy',\ w_3 = y$

$w_1^n = x^2 y''$

$v = x^2,\ v^{(1)} = 2x,\ v^{(2)} = 2,\ v^{(3)} = 0$

$u = y'',\ u^{(1)} = y^{(3)},\ u^{(2)} = y^{(4)},\ u^{(3)} = y^{(5)}$

$u^n = y^{n+2}$

$u^n v^0 + n u^{n-1} v^{(1)} + \frac{n(n-1) u^{n-2} v^{(2)}}{2!}$

$y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1)}{\cancel{2!}} y^n \cdot \cancel{2}$

$w_1^n = \cancel{x^2 y^{n+2} + n y^{n+1} \cdot 2x y^n n(n-1)}$, $x^2 y^{n+2} + 2xn y^{n+1} + n(n-1) y^n$

$w_2^n = xy'$

$u = y',\ u^{(1)} = y^{(2)},\ u^{(2)} = y^{(3)}$

$v = x,\ v^{(1)} = 1,\ v^{(2)} = 0$

$u^n = y^{n+1}$

$y_2^n = U^n V^0 + n U^{n-1} V^{(1)}$

$= y^{n+1} \cdot x + n y^n \cdot 1$

$W_n^2 = x y^{n+1} + n y^n$

$w^3 = y$

$V = 1 \quad , \quad V^{(1)} = 0$

$u = y \quad , \quad u^{(1)} = y^{(1)}$

$u^n = y^n$

$y^n = U^n V^0$

$= y^n \cdot 1$

$W_n^3 = y^n$

$y = w_1 + w_2 + w_3$

$y = x^2 y^{n+2} + \cancel{ny^{n+1}}\, 2xny^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n$

$y^* = x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n$