Odusola Olamilekan Olasubomi

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Chemical engineering.

Eng 361

Assignment 2

① If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$. And hence prove that:

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$.

SOL

$y = e^{x^2+x}$ ——(i) $\frac{dy}{dx} = y' = (2x+1) \cdot e^{x^2+x}$ ——(ii)

$\frac{d^2y}{dx^2} = y'' = \left[(2x+1)\cdot(2x+1)e^{x^2+x}\right] + e^{x^2+x} \cdot 2$ ——(iii)

Sub equ (i) and (ii) into equ (iii) we have:

$y'' = (2x+1)y' + 2y$

If $w_1 = y''$

$v = 1, \quad v^{(1)} = 0$

$u = y'', \quad u' = y''' \quad \therefore u^n = y^{n+2}$

$w_1^n = u^n v + n u^{n-1} v^{(1)}$

$w_1^n = y^{n+2} \cdot 1 + \cancel{n u^{n+1} \cdot 0} = y^{n+2}$

If $w_2 = (2x+1)y' \quad \therefore \quad v = 2x+1, \quad v^{(1)} = 2, \quad v^{(2)} = 0$

$u = y', \quad \therefore u^n = y^{n+1}$

$w_2^n = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$

$= y^{n+1} \cdot (2x+1) + n y^n \cdot 2 + \cancel{\frac{n(n-1)}{2} y^{n-1} \cdot 0}$

$w_2^n = (2x+1)y^{n+1} + 2ny^n$

If $w_3 = 2y \quad \therefore \quad v = 2, \quad v^{(1)} = 0$

$u = y, \quad u^n = y^n$

$w_3^n = u^n v + n u^{n-1} v^{(1)}$

$= y^n \cdot 2 + \cancel{n y^{n-1} \cdot 0} = 2y^n$

$w^n = w_1^n + w_2^n$

(2) Using Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

SOL

$y = x^3 e^{4x}$ $\therefore$ $V = x^3$, $V^{(1)} = 3x^2$, $V^{(2)} = 6x$, $V^{(3)} = 6$, $V^{(4)} = 0$

$U = e^{4x}$, $U^{(1)} = 4e^{4x}$, $U^{(2)} = 16e^{4x}$, $U^{3} = 64e^{4x}$

$\therefore U^n = 4^n e^{4x}$

$$y^n = U^n V + n U^{n-1} V^{(1)} + \frac{n(n-1)}{2} U^{n-2} V^{(2)} + \frac{n(n-1)(n-2)}{3 \cdot 2} U^{n-3} V^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!} U^{n-4} V^{(4)}.$$

$$y^n = 4^n e^{4x} \cdot x^3 + n\, 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3 \cdot 2} 4^{n-3} e^{4x} \cdot 6 \cdot + \cancel{\frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} e^{4x} \cdot 0}.$$

$$y^n = x^3 4^n e^{4x} + n\, 3x^2 4^{n-1} e^{4x} + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^n = 4^{n-3} e^{4x} \left[4^3 x^3 + n 4^2 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2) \right]$$

$\therefore$
$$y^{(5)} = 4^{5-3} e^{4x} \left[4^3 x^3 + 5 \cdot 4^2 \cdot 3x^2 + 5(5-1) 4 \cdot 3x + 5(5-1)(5-2) \right]$$

$$y^{(5)} = 16 e^{4x} \left[64x^3 + 240x^2 + 240x + 60 \right] //$$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2+1) y^{(n)} = 0$

SOL

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0 \Rightarrow x^2 y'' + xy' + y = 0$

If $w_1 = x^2 y''$ $\therefore$ $V = x^2$, $V^{(1)} = 2x$, $V^{(2)} = 2$, $V^{(3)} = 0$

$U = y''$, $U^{(1)} = y'''$ $\therefore U^n = y^{n+2}$

$$w_1^n = U^n V + n U^{n-1} V^{(1)} + \frac{n(n-1)}{2} U^{n-2} V^{(2)} + \frac{n(n-1)(n-2)}{3 \cdot 2} U^{n-3} V^{(3)}$$

$$= y^{n+2} \cdot x^2 + n\, y^{n+1} \cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3 \times 2} y^{n-1} \cdot 0}.$$

$$= x^2 y^{n+2} + 2xn\, y^{n+1} + n(n-1) y^n$$

If $\omega_2 = x y'$ $\therefore$ $v = x$, $v^{(1)} = 1$, $v^{(2)} = 0$

$u = y'$, $u^{(1)} = y''$ $\therefore$ $u^{n} = y^{n+1}$

$$\omega_2^{n} = u^{n} v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$$

$$= y^{n+1} \cdot x + n y^{n} \cdot 1 + \cancel{\frac{n(n-1)}{2} y^{n-1} \cdot 0}$$

$$= x y^{n+1} + n y^{n}$$

If $\omega_3 = y$ $\therefore$ $v = 1$ $v^{(1)} = 0$

$u = y$, $u^{n} = y^{n}$

$$\omega_3^{n} = u^{n} v + n u^{n-1} v^{(1)}$$

$$= y^{n} \cdot 1 + \cancel{n y^{n-1} \cdot 0} = y^{n}$$

$\therefore$ $\omega_1^{n} + \omega_2^{n} + \omega_3^{n} = 0$

$$\Rightarrow x^2 y^{n+2} + 2xn\, y^{n+1} + n(n-1) y^{n} + x y^{n+1} + n y^{n} + y^{n}$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - n + n + 1) y^{n}$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$