Name: ALEGBELEYE OLUWAFEMI OLADIPUPO
Matric No: 17/ENG04/011
Department: ELECT/ELECT

ENG 381 ASSIGNMENT 2

If $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$

and hence prove that

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + x(n+1)\, y^{n}$

Soln

$y = e^{x^2+x}$

Differentiating using function of function

$u = x^2 + x \qquad y = e^u$

$\frac{du}{dx} = 2x+1 \qquad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$dy/dx = (2x+1) \cdot e^u$

$y' = e^{(x^2+x)} \cdot (2x+1)$

But $y' = dy/dx$

$y' = e^{(x^2+x)} \cdot (2x+1)$

Differentiating using product rule

$y'' = \frac{d}{dx}\left[(2x+1)\, e^{x^2+x}\right] = d^2y/dx^2$

$\frac{d^2y}{dx^2} = u\, dv/dx + v\, du/dx.$

Let $u = 2x+1$

$\frac{du}{dx} = 2$

$v = e^{x^2+x}$

Differentiating v using function of function

Let $z = x^2 + x$, $dz/dx = 2x+1$

$v = e^z$; $dv/dz = e^z$

$dv/dx = (2x+1) \cdot e^{x^2+x}$

Substituting them into product formula

$y'' = v\, du/dx + u\, dv/dx$

$y'' = e^{x^2+x} \cdot 2 + 2x+1 \cdot (2x+1)\, e^{x^2+x}$

$y'' = 2e^{x^2+x} + 2x+1 \cdot (2x+1)\, e^{x^2+x}$

But $y = e^{x^2+x}$; $y' = (2x+1)\, e^{x^2+x}$

$\therefore \quad y'' = y'(2x+1) + 2y$ ——— (*)

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from eqn (*) above

$y'(2x+1) + 2y - y'' = 0$

let $w_1 = y'(2x+1)$

$w_2 = 2y$

$w_3 = -y''$

$w_1 = y'(2x+1)$

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v^2 = 0$

Using Lebnitz theorem

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots\ldots$

Substituting

$y^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2 + 0$

$y^n = y^{(n+1)} \cdot (2x+1) + 2ny^n$

$w_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

Using Lebnitz theorem

$y^n = y^n \cdot 2$; $y^n = 2y^n$

$w_3 = -y^2$

$u = y^2$ $\quad v = -1$

$u^n = y^{n+2}$ $\quad v = 0$

Using Lebnitz theorem

$y^n = y^{n+2} \cdot -1$

$y^n = -y^{n+2}$

Summing all together

$y^n = y^{(n+1)} \cdot 2x+1 + 2ny^n + 2y^n - y^{n+2} = 0$

Collecting like term.

$y^{n+2} = y^{n+1} \cdot (2x+1) + 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n = 0$

Question 2.

Using Lebnitz theorem, given that

$y = x^3 e^{4x}$

Soln

$y = x^3 \cdot e^{4x}$

$U = e^{4x}$ $\quad V = x^3$

$U^n = 4^n e^{4x}$ $\quad V' = 3x^2$

$U^{n-1} = 4^{(n-1)} e^{4x}$ $\quad V^2 = 6x$

$U^{n-2} = 4^{(n-2)} e^{4x}$ $\quad V^3 = 6$

$U^{n-3} = 4^{(n-3)} e^{4x}$ $\quad V^4 = 0$

Using Leibnitz theorem.

$$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \ldots$$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)}e^{4x} \cdot 3x^2 + \frac{(n)(n-1)}{2!}4^{n-2}e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!}4^{(n-3)}e^{4x} \cdot 6 + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)}e^{4x} \cdot 3x^2 + n^2 - n \cdot 4^{n-2}e^{4x} \cdot 3x + n(n^2-3n+2) \cdot 4^{n-3}e^{4x} \cdot 2$$

factorizing e^{4x}

$$y^n = e^{4x}\left[(4^n \cdot x^3) + (n4^{n-1} \cdot 3x^2) + (n^2-n\,4^{n-2} \cdot 3x) + (n^3-3n^2+2n\,8^{n-3})\right]$$

factorizing 4^{n-3}

$$y^n = e^{4x}\left[(4^{n-n+3} \cdot x^3) + (n4^{(n-1-n+3)} \cdot 3x^2) + (n^2-n4^{n-2-n+3} \cdot 3x) + (n^3-3n^2+2n\,8^{n-3-n+3})\right]$$

$$y^n = e^{4x}4^{n-3}\left[(4^3 \cdot x^3) + (n4^2 \cdot 3x^2) + (n^2-n \cdot 4^1 \cdot 3x) + (n^3-3n^2+2n\,8^0)\right]$$

But $n = 5$

~~$y^5 = e^{4x}[64x^3 +$~~

$$y^5 = e^{4x} \cdot 4^{n-3}\left[64x^3 + 48nx^2 + 12xn^2 - n + n^3 - 3n^2 + 2n\right]$$

$$y^5 = e^{4x}4^2\left[64x^3 + 240x^2 + 240x + 60\right]$$

$$y^5 = 16e^{4x}\left[64x^3 + 240x^2 + 240x + 60\right]$$

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11) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

rewritting equation

$x^2y'' + xy' + y = 0$

for $w_1 = x^2y''$ $\quad W_2 = xy'$ $\quad W_3 = y$

for w_1

$U = y^2 \qquad V = x^2$

$U^n = y^{n+2} \qquad V' = 2x$

$U^{n-1} = y^{n+1} \qquad V^2 = 2$

$U^{n-2} = y^n \qquad V^3 = 0$

for w_2

$U = y' \qquad V = x$

$U^n = y^{n+1} \qquad V' = 1$

$U^{n-1} = y^n \qquad V^2 = 0$

for w_3

$U = y \qquad V = 1$

$U^n = y^n \qquad V' = 0$

Using Lebnitz theorem

$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \cdots$

for w_1

$y^n = y^{(n+2)}x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2}y^n \cdot 2 + 0$

for w_2

$y^n = y^{(n+1)}\cdot x + n\cdot y^n \cdot 1 + 0$

for w_3

$y^n = y^n \cdot 1 + 0$

Summing together:

$y^n = x^2y^{(n+2)} + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{\cancel{2}}y^n\cdot\cancel{2} + y^{(n+1)}\cdot x + ny^n + y^n = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$y^n = x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$