Mechanical engineering

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

from the above equation

Part A, $A = y''$, $A' = y'''$, $A^n = y^{2+n}$

Part B, $B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$, $v' = 2$

$v''' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + 2n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

Part C, $C = 2y$, $C^n = 2y^n$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) $y = x^3 e^{4x}$,

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$

$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

using leibnitz theorem

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} x + 5(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$$

$$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

(2b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Part A, $A = x^2y''$

$u = y''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$A^n = (y^{n+2})x^2 + n(y^{n+1})\cdot 2x + \frac{n(n-1)}{2!}\cdot(y^n)\cdot 2 + 0$

$A^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

Part B, $B = xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$B^n = (y^{n+1})\cdot x + n(y^n)\cdot 1 + 0$

$= xy^{(n+1)} + ny^n$

Part C, $c = y$, $c^n = y^n$

$\therefore A^n + B^n + C^n = 0$

$\Rightarrow x^2y^{(n+2)} + 2x\cancel{n}y^{(n+1)} + (n^2-n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$\Rightarrow x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2-n+n+1) = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$