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ENG 381 [ENGINEERING MATHEMATICS] ASSIGNMENT 2

1.) $y = e^{x^2+x}$

Show that
$y'' = y'(2x+1) + 2y$

and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

Soln

$y = e^{x^2+x}$

Differentiate $y = y'$ & y''

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$

$y'' = (2x+1)y' + y \cdot 2$

∴ Therefore,

$y'' = y'(2x+1) + 2y$

Finding the nth derivative of y''

i.e $y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)} \cdot 2 + 2y^{(n)}$.

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

Therefore

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$.

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$.

2.)

$y = x^3 e^{4x}$, determine $y^{(5)}$

Soln

$$y^n = U^n V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V''' + \frac{n(n-1)(n-2)(n-3)}{3!}U^{(n-4)}V^{(4)} + \dots$$

taking

$U = e^{4x}$ $\quad$ $V = x^3$

$U^n = 4^n e^{4x}$ $\quad$ $V' = 3x^2$

$U^{(n-1)} = 4^{(n-1)}e^{4x}$ $\quad$ $V'' = 6x$

$U^{(n-2)} = 4^{(n-2)}e^{4x}$ $\quad$ $V''' = 6$

$U^{(n-3)} = 4^{(n-3)}e^{4x}$ $\quad$ $V^{(4)} = 0$

$U^{(n-4)} = 4^{(n-4)}e^{4x}$ $\quad$ $V^{(5)} = 0$

$U^{(n-5)} = 4^{(n-5)}e^{4x}$

Sub values in General soln

$$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)}e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)}e^{4x} \cdot 6x + \dots$$

$$\dots + \frac{n(n-1)(n-2)}{3!}4^{(n-3)}e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!}4^{(n-4)}e^{4x} \cdot 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \frac{n(n-1)}{2\times 1}6x \cdot 4^{(n-2)}e^{4x} + \frac{n(n-1)(n-2)}{3\times 2\times 1} \cdot 6 \cdot 4^{(n-3)}e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \frac{n(n-1)}{\cancel{2}1}\overset{3}{\cancel{6}}x \cdot 4^{(n-2)}e^{4x} + \frac{\cancel{6}n(n-1)(n-2)}{\cancel{6}} \cdot 4^{(n-3)}e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \overset{3nx}{\cancel{3nx}}(n-1)4^{(n-2)}e^{4x} + n(n-1)(n-2)4^{(n-3)}e^{4x} + 0$$

$y^n = y^5$, i.e $n = 5$, Sub value $n=5$ into eqn above

$$y^5 = x^3 4^{(5)}e^{4x} + 3(5)x^2 4^{(5-1)}e^{4x} + \overset{3(5)(5-1)x}{\cancel{3(5)(5-1)}}4^{(5-2)}e^{4x} + 5(5-1)(5-2)4^{(5-3)}e^{4x} + 0$$

$$15(4)(64)\,x\,e^{4x}$$

$$y^5 = 1024x^3e^{4x} + 15(256)x^2e^{4x} + \cancel{15(4)(64)e^{4x}} + \; + 5(4)(3)(16)e^{4x} + 0$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$$

(2ii)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Show that

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Soln.

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2y'' + xy' + y = 0$$

Taking $a_1 = x^2y''$, $a_2 = xy'$, $a_3 = y$

Considering; a_1 $\quad a_2 \quad a_3$

$U = y''$	$V = x^2$	$U = y'$	$V = x$	$U = y$	$V = 1$
$U^{(n)} = y^{(n+2)}$	$V' = 2x$	$U^{(n)} = y^{(n+1)}$	$V' = 1$	$U^n = y^n$	$V' = 0$
$U^{(n-1)} = y^{(n+1)}$	$V'' = 2$	$U^{(n-1)} = y^n$	$V'' = 0$		
$U^{(n-2)} = y^{(n)}$	$V''' = 0$				

$$y^{(n)} = U^{(n)}V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V''' + \ldots$$

$$\ldots + \frac{n(n-1)(n-2)(n-3)}{4!}U^{(n-4)}V^{(4)} + \ldots$$

Therefore;

$$a_1^{(n)} = y^{(n+2)}\cdot x^2 + n\left[y^{(n+1)}\cdot 2x\right) + \frac{n(n-1)}{2!}\cdot y^n\cdot 2 + 0$$

$$a_2^{(n)} = y^{(n+1)}\cdot x + n\left(y^n\cdot 1\right] + 0$$

$$a_3^{(n)} = y^n\cdot 1 + 0$$

Substitute Values of $a_1^{(n)}, a_2^{(n)}, a_3^{(n)}$ into general eqn

$$x^2y'' + xy' + y = 0$$

$$y^{(n+2)} \cdot x^2 + 2nx\, y^{(n+1)} + \frac{n(n-1)}{2\times 1} \cdot 2\, y^n + y^{(n+1)} \cdot x + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + \frac{n(n-1)}{\cancel{2}} \cdot \cancel{2}\, y^n + x\, y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + n(n-1)\, y^n + x\, y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + x\, y^{(n+1)} + n(n-1)\, y^n + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1)\, x\, y^{(n+1)} + (n^2 - \cancel{n} + \cancel{n} + 1]\, y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1)\, x\, y^{(n+1)} + [n^2 + 1]\, y^n = 0$$