- Name: Usman Gift Amira.
- department: Chemical Engineering.
- Matric NO: 17/ENG01/081.

Engineering Mathematics Assignment 2.

① $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Solution.

$y = e^{x^2+x}$

Differentiating.

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}.2$

$y'' = (2x+1)y' + 2y$

Therefore $y'' = y'(2x+1) + 2y$

finding the nth derivative of y''

$w_1 = (2x+1)y'$

$w_2 = 2y$

for w_1

$u = y'$ $\quad v = 2x+1$

$u^n = y^{(n+1)}$ $\quad v' = 2$

$u^{(n-1)} = y^n$ $\quad v'' = 0$

for w_2.

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

rearranging $w_1 = y^{(n+1)}(2x+1) + ny^n(2)$

" $w_2 = y^n(2)$

putting in altogether back into the Equation

$= y^{(n+1)}(2x+1) + ny^{(n)}2 + y^n(2)$

rearranging.

$= y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$= y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

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Therefore

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1) \quad y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

(2) $y = x^3 e^{4x}$, determine $y^{(5)}$

$$y^{(n)} = U^n V + nU^{(n-1)}V' + \frac{n(n-1)U^{(n-2)}V''}{2!} + \frac{n(n-1)(n-2)U^{(n-3)}V'''}{3!} + \frac{n(n-1)(n-2)(n-3)U^{(n-4)}V^{(4)}}{4!} +$$

Assume:

$U = e^{4x}$	$V = x^3$
$U^n = 4^n e^{4x}$	$V' = 3x^2$
$U^{(n-1)} = 4^{(n-1)} e^{4x}$	$V'' = 6x$
$U^{(n-2)} = 4^{(n-2)} e^{4x}$	$V''' = 6$
$U^{(n-3)} = 4^{(n-3)} e^{4x}$	$V^{iv} = 0$
$U^{(n-4)} = 4^{(n-4)} e^{4x}$	$V^{v} = 0$
$U^{(n-5)} = 4^{(n-5)} e^{4x}$	

Substituting the value above in the general solution.

$$y^n = 4^n e^{4x} \cdot x^3 + n\left[4^{(n-1)} e^{4x} \cdot 3x^2\right] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{(n-3)} e^{4x} \cdot 6$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} \cdot 4^{(n-4)} e^{4x} \cdot 0 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \cdot 4^{(n-5)} e^{4x} \cdot 0$$

$$y^{(n)} = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)^3 6x}{2!} \cdot 4^{(n-2)} e^{4x} + \frac{6n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x}$$

$+ 0 + 0 \ldots\ldots$

$$y^{(n)} = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$$

Recall:

$y^n = y^5$ where n is 5,

Substituting $n=5$ into the equation above

$$y^{(5)} = x^3 4^{(5)} e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x} + 3(5)x(5-1) 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x} + 0.$$

$$y^{(5)} = 1024x^3 e^{4x} + 15(256)x^2 e^{4x} + 15(256)xe^{4x} + 60(16)e^{4x} + 0$$

$$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$$

2ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that
$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

solution:

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Recall $\frac{d^2y}{dx^2} = y''$ $\quad \frac{dy}{dx} = y'$

$x^2y'' + xy' + y = 0$

$w_1 = x^2y'', \ w_2 = xy', \ w_3 = y$

w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{(n+2)}$ $\quad v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^n$ $\quad v''' = 0$

w_2

$u = y$, $\quad v = x$

$u^n = y^{(n+1)}$ $\quad v' = 1$

$u^{(n-1)} = y^n$ $\quad v'' = 0$

w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v''' + n(n-1)(n-2)(n-3)u^{(n}$
$+ \ldots$

$w_1 = y^{(n+2)}\cdot x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}y^{(n)}\cdot 2 + 0$

$w_2 = y^{(n+1)}\cdot x + ny^n \cdot 1 + 0$

$w_3 = y^n + 0$

applying leibnitz theorem

$= x^2y^{(n+2)} + 2xny^{(n+1)} + ny^{(n)}(n-1) + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}[n^2 - \cancel{x} + \cancel{x} + 1] = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2+1)y^{n} = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{n} = 0.$

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