Assignment 1

If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Sol

$y = e^{x^2+x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$$

$y' = (2x+1)e^{x^2+x}$ $\quad u = 2x+1 \quad du/dx = 2 \quad v = e^{x^2+x} \quad dv/dx = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$

$y'' = (2x+1)y' + 2y$

$y'' = y^{(n+2)}$

$y'(2x+1) = (2x+1)y^{(n+1)} + 2ny^n$

$2y = 2y^n$

$\therefore \quad y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

Assignment 2

- $y = x^3 e^{4x}$, find $y^{(5)}$ $\quad u = e^{4x} \quad v = x^3$

Sol $y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(5-1)} \cdot 3x^2 + \frac{5(5-1)}{2!} 4^{(5-2)} e^{4x} \cdot 6x +$$

$$\frac{5(5-1)(5-2)}{3!} 4^{(5-3)} e^{4x} \cdot 6 + 0$$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960$

$y^5 = 64(16e^{4x}x^3 + 60e^{4x}x^2 + 60e^{4x}x + 15)$

$x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Sol: $x^2 y'' + x y' + y = 0$

$\quad A \qquad\quad B \qquad C$

$A: \; x^2 y'' = x^2 y^{(n+2)} + 2xn y^{(n+1)} + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$= x^2 y^{(n+2)} + 2xn y^{(n+1)} + (n^2 - n) y^n$

$B: \; x y' = x y^{(n+1)} + n y^n$

$C: \; y = y^n$

$\therefore \; A + B + C = 0$

$\Rightarrow x^2 y^{(n+2)} + 2xn y^{(n+1)} + (n^2 - n) y^n + x y^{(n+1)} + n y^n + y^n = 0$

$\Rightarrow x^2 y^{(n+2)} + 2xn y^{(n+1)} + x y^{(n+1)} + (n^2 - n) y^n + n y^n + y^n = 0$

$\Rightarrow x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + y^n (n^2 \cancel{-n} \cancel{+n} + 1) = 0$

$\Rightarrow x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + y^n (n^2 + 1) = 0$