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17/ENG02/063

COMPUTER ENGINEERING

Question 1

$$y'' = y'(2x+1) + 2y \quad --- (*)$$

1. $y = e^{x^2+x}$

let $u = x^2 + x$ $\qquad \frac{du}{dx} = 2x + 1$

$y = e^u$, $\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \cdot (2x+1)$

$$\frac{dy}{dx} = \underset{R}{\underset{\downarrow}{e^{(x^2+x)}}} \quad \underset{S}{\underset{\downarrow}{(2x+1)}}$$

$$\frac{d^2y}{dx^2} = \frac{R\,dS}{dx} + \frac{S\,dR}{dx}$$

$= e^{x^2+x}(2) + (2x+1)\left[(2x+1) \cdot e^{x^2+x}\right]$

$= 2e^{x^2+x} + (4x^2+4x+1)(e^{x^2+x})$

$\rightarrow \frac{dy}{dx}(2x+1) + 2y$

$\left[(2x+1)(e^{x^2+x})\right](2x+1) + 2e^{x^2+x}$

$(4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

Since $\frac{d^2y}{dx^2} = 2e^{x^2+x} + (4x^2+4x+1)e^{x^2+x}$

AND

$\frac{dy}{dx}(2x+1) + 2y = 2e^{x^2+x} + (4x^2+4x+1)e^{x^2+x}$,

$$\therefore \frac{d^2y}{dx^2} = \frac{dy}{dx}(2x+1) + 2y$$

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To prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Take the R·H·S of eqn (*)

$$y'(2x+1) + 2y$$

$$W_1 = y'(2x+1) \quad ; \quad W_2 = 2y$$

$$W_1^{(n)} = y^{(n+1)}(2x+1) + n\,y^{(n)}\,2$$

$$W_2^{(n)} = 2y^{(n)} + 0$$

$$\Rightarrow (2x+1)y^{(n+1)} + (2n+2)y^{(n)}$$

$$(2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

$$\therefore y'(2x+1) + 2y = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

$$y'' = \frac{d^2y}{dx^2}$$

$$W_3 = \frac{d^2y}{dx^2} \quad ; \quad W_3^{n} = \left(\frac{d^2y}{dx^2}\right)^{n} = \frac{d^{(n+2)}y}{dx^{(n+2)}} = y^{(n+2)}$$

$$\therefore y'' \equiv y^{(n+2)}$$

Since $\cancel{y''} \equiv y'(2x+1) + 2y \equiv (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

and $y'' = y^{(n+2)}$

$$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$$

because $y'' = y'(2x+1) + 2y$.

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Question 2.

i $y = x^3 e^{4x}$ $\qquad$ $y^{(5)} = ?$

$v = x^3$

$u = e^{4x}$

using the leibnitz theorem:

$$y^{(5)} = u^{(5)}v + 5u^{(5-1)}v^{(1)} + \frac{5(5-1)}{2!}u^{(5-2)}v^{(2)}$$

$$+ \frac{5(5-1)(5-2)}{3!}u^{(5-3)}v^{(3)} + \frac{5(5-1)(5-2)(5-3)}{4!}u^{(5-4)}v^{(4)}$$

$$+ \frac{5(5-1)(5-2)(5-3)(5-4)}{5!}u^{(5-5)}v^{(5)}$$

$v = x^3$; $v^{(1)} = 3x^2$; $v^{(2)} = 6x$; $v^{(3)} = 6$; $v^{(4)} = 0$

$v^{(5)} = 0$

$u = e^{4x}$; $u^{(5)} = 1024e^{4x}$; $u^{(4)} = 256e^{4x}$

$u^{(3)} = 64e^{4x}$; $u^{(2)} = 16e^{4x}$; $u^{(1)} = 4e^{4x}$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x$$
$$+ 960e^{4x} + 0$$

ii) $\left(\frac{d^2y}{dx^2}\right)^{(n)} = \frac{d^{(n+2)}y}{dx^{(n+2)}}$

$$\left(\frac{d^2y}{dx^2}\right)^{(n-2)} = \cancel{y^{(n)}}\ y^{(n)}$$

$$\left(y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^{(n)}2\right) + \left(y^{(n+1)}x + ny^{(n)}(1) + 0\right) + \left(y^{(n)}\right) = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + \left(n(n+1) + n + 1\right)y^{(n)} = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$