BRIGGS FRANCIS SOIBI 17/ENG04/015

1) $y = e^{x^2+x}$

$y^{(1)} = (2x+1)e^{x^2+x}$

$y^{(11)} = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\Rightarrow y^{(11)} = 2y + y'(2x+1)$

ii prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$\omega_1 = \cancel{y^{(n+2)}}\ y^{(2)}$

$\omega_2 = y'(2x+1)$

$\omega_3 = 2y$

for ω_1

let $y^{(2)} = F$, $F' = y^{(2+1)} \Rightarrow F^n \Rightarrow y^{2+n}$

for ω_2

let $y^{(1)}(2x+1) = S$

let $u = y^{(1)}$ $v = 2x+1$

$u' = y^{(2)}$ $v^{(1)} = 2$

$u'' = y^{(3)}$ $v^{(11)} = 0$

$\Rightarrow u^{(n)} = y^{(1+n)}$ $u^{(n-1)} = y^{(n)}$

from leibnitz

S^n becomes $= (y^{(1+n)})(2x+1) + n(y^n)(2) + 0$

$S^n = (2x+1)y^{(1+n)} + 2ny^n$

for ω_3

$B = 2y$

$B^n = 2y^n$

$F^n = S^n + B^n$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2ny^{(n)} + 2y^{(n)}$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

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2) $y = x^3 \cdot e^{4x}$

$y^{(5)} = ?$

let $u = e^{4x}$ and $v = x^3$

$u^{(1)} = 4e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(11)} = 16e^{4x}$ $\quad v^{(11)} = 6x$

$u^{(111)} = 64e^{4x}$ $\quad v^{(111)} = 6$

$u^{(4)} = 256e^{4x}$ $\quad v^{(4)} = 0$

$u^{(n)} = 4^n e^{4x}$

using leibnitz theom

$y^{(n)} = {}^nC_0 u^{(n)} v^{(0)} + {}^nC_1 u^{(n-1)} v^{(1)} + {}^nC_2 u^{(n-2)} v^{(2)} + {}^nC_3 u^{(n-3)} v^{(3)} + {}^nC_4 u^{(n-4)} v^{(4)}$

$= 4^n e^{4x}(x^3) + \frac{n\cancel{(n-1)!}}{1!\cancel{(n-1)!}} 4^{(n-1)} \cdot 3x^2 \cdot e^{4x} + \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} (4^{(n-2)})(6x)e^{4x} +$

$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} (4^{(n-3)} \cdot e^{4x} \cdot 6) + 0$

$y^{(n)} = 4^n \cdot e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{(n-1)} \cdot e^{4x} + 3n(n-1) \cdot 4^{n-2} \cdot e^{4x} \cdot x + n(n-1)(n-2)4^{n-3} \cdot e^{4x}$

when $n = 5$

$y^{(5)} = 4^5 \cdot e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 \cdot e^{4x} + 3(5)(4) \cdot 4^3 \cdot e^{4x} \cdot x + 5(4)(3)4^2 \cdot e^{4x}$

$y^{(5)} = 1024e^{4x} \cdot x^3 + 3840e^{4x} \cdot x^2 + 3840e^{4x} \cdot x + 960e^{4x}$

$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

ii $\quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2+1)y^n = 0$

$x^2 y^{(11)} + xy^{(1)} + y^{(0)} = 0$

$w_1 = x^2 y^{(11)}$

$w_2 = x y^{(1)}$

$w_3 = y^{(0)}$

for w_1

let $u = y^{(11)}$ and $v = x^2$

$u^n = y^{(n+2)}$ $\quad v^{(1)} = 2x$ $\quad v^{(11)} = 2$ $\quad v^{(111)} = 0$

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using leibintz

$^nC_0 u^{(n)} v^{(0)} + {}^nC_1 u^{(n-1)} v^{(1)} + {}^nC_2 u^{(n-2)} v^{(2)} + {}^nC_3 u^{(n-3)} v^{(3)} + \ldots\ldots$

$\Rightarrow x^2 y^{(n+2)} + \frac{2xn\cancel{(n-1)!}}{1!\cancel{(n-1)!}} y^{(n+1)} + \frac{2n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} \cdot y^n$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

for part w_2

$w_2 = x y^{(1)}$

$u = y^{(1)} \Rightarrow u^n = y^{(1+n)}$

$v = x \quad v^{(1)} = 1 \quad v'' = 0$

using leibnitz

$xy^{(n+1)} + ny^n + 0$

for w_3

$u = y \quad v = 1$

$u^{(n)} = y^{(n)}$

$\Rightarrow$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^n + xy^{n+1} + ny^n + y^n = 0$

$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2-n+n+1) = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

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