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17/ENG06/055
Mechanical Engineering
ENG 381

1) If $y = e^{x^2+x}$
$y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y = e^{x^2+x}$
$y' = 2x+1\,(e^{x^2+x})$
$y'' = 2(e^{x^2+x}) + (2x+1)(2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$
$y'' = y'(2x+1) + 2(y)$

$A = y''$ $\quad A_n = y^{n+2}$
$B = y'(2x+1)$ $\quad B_n = y^{(n+1)}(2x+1) + ny^n(2)$
$C = 2y$ $\quad C_n = 2y^n$
$\therefore \quad y^{n+2} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$
$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2) Using lebnitz theorem
i) $y = x^3e^{4x}$ determine y^5
$V = x^3$ $\quad V' = 3x^2$ $\quad V'' = 6x$ $\quad V''' = 6$ $\quad V^n = \cancel{x^{n+3}}$
$U = e^{4x}$ $\quad U' = 4e^{4x}$ $\quad U'' = 16e^{4x}$ $\quad U''' = 4^ne^{4x}$
$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)U^{n-2}V''}{2!} + \dots$
$y^n = 4^ne^{4x}\cdot x^3 + n\cdot 4^{n-1}e^{4x}\cdot 3x^2 + \frac{n(n-1)4^{n-2}e^{4x}\cdot 6x}{2!} +$
$\frac{n(n-1)(n-2)}{3!}\cdot 4^{n-3}e^{4x}6 + 0$
$y^n = 4^ne^{4x}\cdot x^3 + 3x^2n4^{n-1}e^{4x} + 3n(n-4)4^{n-2}e^{4x}x +$
$n(n-1)(n-2)\cdot 4^{n-3}e^{4x}$
$y^5 = 4^5e^{4x}\cdot x^3 + 3x^2(5)4^{5-1}e^{4x} + 3(5)(5-1)4^{(5-2)}e^{4x}x +$
$5(5-1)(5-2)\cdot 4^{(5-3)}e^{4x}$
$y^5 = 1024e^{4x}\cdot x^3 + 3840x^2e^{4x} + 3840e^{4x}\cdot x + 960e^{4x}$

$\therefore y^5 = \cancel{64e} \; 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution

$= x^2y'' + xy' + y = 0$

$\cancel{A^n = y^{n+2}x^2 +}$

$A = x^2y''$, $A^n = y^{n+2}x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)\cancel{(n+2)}y^n \cdot \cancel{2}}{\cancel{2}!} + 0$

$A^n = y^{n+2}x^2 + ny^{n+1} \cdot 2x + n(n-1)(n-2)y^n$

$B = xy'$ $\quad B^n = y^{n+1}x + ny^n$

$C = y$ $\quad C_n = y^n$

$\therefore$

$= y^{n+2}x^2 + ny^{n+1}2x + (n^2-n)y^n + y^{n+1}x + ny^n + y^n$

$\cancel{2x} \; x^2(y^{n+2}) + xy^{n+1}(2n+1) + y^n(n^2-n+n+1)$

$= x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$