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17/ENG05/030

MECHATRONICS ENGINEERING

1) If $y = e^{x^2+x}$

Differentiating

$y = e^u$

Let $u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$dy/du = e^u$

Differentiating using function of function

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u(2x+1) = (2x+1)e^{x^2+x}$

But $y' = \frac{dy}{dx}$

$y' = (2x+1)e^{x^2+x}$

Differentiating further

$\frac{d^2y}{dx^2} = y'' = \frac{d}{dx}(2x+1)e^{x^2+x}$

Let $u = 2x+1 \Rightarrow \frac{du}{dx} = 2$

$V = e^{x^2+x}$

Differentiating using function of function

$dV/dx = dV/dz \times dz/dx$

Let $z = x^2+x$

$dz/dx = 2x+1$ $\quad \frac{dv}{dz} = e^z = e^{x^2+x}$

$\frac{dv}{dx} = e^z(2x+1) = e^{(x^2+x)}(2x+1)$

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$

Differentiating using Product Rule

$\frac{d^2y}{dx^2} = u\frac{dv}{dz} + v\frac{du}{dx}$

$= (2x+1)[(2x+1)e^{x^2+x}] + e^{(x^2+x)} \times 2$

$= (2x+1)[(2x+1)e^{x^2+x}] + 2e^{x^2+x}$

But $y = e^{x^2+x}$ and $y' = (2x+1)\, e^{x^2+x}$

$$\frac{d^2y}{dx^2} = y'(2x+1) + 2y$$

Since $\frac{d^2y}{dx^2} = y''$

$$y'' = y'(2x+1) + 2y$$

$$y'(2x+1) + 2y - y'' = 0$$

Let $W_1 = y'(2x+1)$

$W_2 = 2y$

$W_3 = y''$

For W_1

$u = y'$, $v = 2x+1$

$u^{(n)} = y^{(n+1)}$, $v' = 2$

$u^{(n-1)} = y^{(n)}$, $v'' = 0$

For W_2

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

For W_3

$u = y''$, $v = 1$

$u^n = y^{(n+2)}$, $v = 0$

$u^{(n-1)} = y^{(n+1)}$

Recall

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

$$= 2x+1 \cdot y^{(n+1)} + 2n y^n + 2y^n + 2y^n + y^{(n+2)} \neq 0$$

$$y^{(n+2)} = (2x+1) \cdot y^{(n+1)} + 2n y^n + 2y^n$$

$$y^{(n+2)} = (2x+1) \cdot y^{(n+1)} + 2(n+1) y^n$$

Question 2:

$y = x^3 e^{4x}$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(n-5)} v^5 + \dots$$

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$y = e^{4x}$

$y^{n} = 4^{n} e^{4x}$

$y^{(n-1)} = 4^{(n-1)} e^{4x}$

$y^{(n-2)} = 4^{(n-2)} e^{4x}$

$y^{(n-3)} = 4^{(n-3)} e^{4x}$

$V = x^3$

$V^{(1)} = 3x^2$

$V^{(2)} = 6x$

$V^{(3)} = 6$

$V^{(4)} = 0$

$$y^{n} = 4^{n} e^{4x} \cdot x^3 + \cancel{n4} \, n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6$$

$$\cancel{y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + 5[4^{(5-1)} e^{4x} \cdot 3x^2] +}$$

$$y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + 5[4^{(4)} \cdot 3x^2 e^{4x}] + \frac{5(4)}{2!} 4^3 \cdot 6x e^{4x} + \frac{5(4)(3)}{3!} 4^{(2)} e^{4x} \cdot 6$$

$$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$$

$$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$$

Question 2ii

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

Let

$W_1 = x^2 y''$

$W_2 = x y'$

$W_3 = y$

$W_1 = x^2 y''$

$u = y^2$

$u^{n} = y^{(n+2)}$

$u^{(n-1)} = y^{(n+1)}$

$u^{(n-2)} = y^{n}$

$V = x^2$

$V^1 = 2x$

$V^2 = 2$

$V^3 = 0$

$W_2 = x y'$

$u = y'$

$u^{n} = y^{(n+1)}$

$u^{(n-1)} = y^{n}$

$V = x$

$V^1 = 1$

$V^2 = 0$

$W_3 = y$

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

From Leibnitz theorem

$$u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^3 + \dots$$

$$y^{(n+2)} \cdot x^2 + n(y^{(n+1)} \cdot 2x) + \frac{n(n-1)}{2!}y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!}y^n$$

$$x^2 y^{(n+2)} + 2xny^{(n+1)} + \frac{n(n-1)}{2!}y^n \cdot 2 + xy^{(n+1)} + ny^n + y^n = 0$$

$$x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + [n(n-1) + n + 1]y^n = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$