QUESTION 1

1a) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{\delta}{\delta x}(e^{x^2+x}) + e^{x^2+x}\frac{\partial}{\partial x}(2x+1)$ (Product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proven

1b) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$W_1 = y''$

$W_2 = y'(2x+1)$

$W_3 = 2y$

degenerate eqn $= W_1 - W_2 - W_3 = 0$

W_1

$U = y''$ $\quad U' = y'''$ $\quad$ hence $U^n = y^{n+2}$

$V = 1$ $\quad V' = 0$

$W_1 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \ldots\ldots$

$W_1 = y^{n+2}(1) + ny^{n+1}(0)$

$W_1 = y^{n+2}$ ------ (i)

W_2 -

$W_2 = y'(2x+1$

$U = y'$ $\quad U' = y''$ $\quad U'' = y'''$ $\quad$ Hence $U^n = y^{n+1}$

$V = 2x+1$ $\quad V' = 2$ $\quad V'' = 0$

$W_2 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \ldots$

$W_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!}y^{n-2}(0) + \ldots$

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$W_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$W_2 = y^{n+1}(2x+1) + 2ny^n \ldots\ldots$ (ii)

W_3

$U = y \quad U' = y'$ Hence $U^n = y^n$

$V = 2 \quad V' = 0$

$$W_3 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \ldots$$

$W_3 = y^n(2) + ny^{n-1}(0) + \ldots$

$W_3 = 2y^n$ ---------------- (iii)

Putting back into the degenerate eqn

$W_1 - W_2 - W_3 = 0$

$y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \Longrightarrow$ Proven

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QUESTION 2

2a) $y = x^3 e^{4x}$

Using Leibnitz Theorem

$U = e^{4x}$ $U' = 4e^{4x}$ $U'' = 16e^{4x}$ $U''' = 64e^{4x}$ $U^{iv} = 256e^{4x}$

$V = x^3$ $V' = 3x^2$ $V'' = 6x$ $V''' = 6$ $V^{iv} = 0$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^2$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^2 + \dots$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!}4^{n-3}e^{4x}(6x)$$
$$+ \frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}(0)^2$$

$$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1}e^{4x} + 3x(n(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3}e^{4x} + 0$$

$$y^n = e^{4n}4^{n-3}[x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2)]$$

$$y^n = e^{4x}4^{n-3}[64x^3 + 48x^2 n + 12x(n(n-1) + n(n-1)(n-2)]$$

$$y^5 = e^{4x}4^{5-3}[64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2)]$$

$$y^5 = e^{4x}4^2[64x^3 + 240x^2 + 240x + 36]$$

$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

2b $x^2\frac{\delta^2 y}{\delta x^2} + \frac{n\, dy}{dx} + y = 0$

$x^2 y^x + xy' + y = 0$

$W_1 + W_2 + W_3 = 0$

$W_1 = x^2 y''$

$U = y''$ $U' = y'''$ $U'' = y^{iv}$ $U''' = y^v$ Hence $U^n = y^{n+2}$

$V = x^2$ $V' = 2x$ $V'' = 2$ $V''' = 0$

$$W_1 = U^n(V) + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}(V^{iv}) + \frac{n(n-1)(n-2)}{3!}U^{n-3}V''' + \dots$$

$$W_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2!}y^n(2) + 0$$

$$W_1 = x^2 y^{n+2} + 2xny^{n+1} + y^n n(n-1)$$

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$W_2 = xy'$

$u = y'$ $\quad u' = y''$ $\quad u'' = y'''$ $\quad$ Hence $u^n = y^{n+1}$

$v = x$ $\quad v' = 1$ $\quad v'' = 0$

$$W_2 = u^n(v) + nu^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v^2.$$

$W_2 = y^{n+1}(x) + ny^n(1) + 0$

$W_2 = xy^{n+1} + ny^n$

$W_3 = y$

$u = y$ $\quad u' = y'$ $\quad$ Hence $u^n = y^n$

$v = 1$ $\quad v' = 0$

$W_3 = u^n(v) + nu^{n-1}(v')$

$= y(1) + 0 = y$

$W_1 + W_2 + W_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + (n+1))$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$

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