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Petroleum Engineering

Question 1

$$y = e^{x^2+x}$$

Solution

$$y = e^{x^2+x}$$

Differentiating

$$y = e^u$$

$$\text{let } u = x^2+x$$

$$\frac{du}{dx} = 2x+1$$

$$\frac{dy}{du} = e^u$$

Differentiating Using function of function

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u(2x+1) = (2x+1)e^{x^2+x}$$

$$\text{But } y' = \frac{dy}{dx}$$

$$y' = (2x+1)e^{x^2+x}$$

Differentiating further

$$\frac{d^2y}{dx^2} = y'' = \frac{d}{dx} (2x+1)e^{x^2+x}$$

$$\text{let } u = 2x+1 \quad \Rightarrow \quad \frac{du}{dx} = 2$$

$$v = e^{x^2+x}$$

Differentiating Using function of function

$$\frac{dv}{dx} = \frac{dv}{dz} \times \frac{dz}{dx}$$

$$\text{let } z = x^2+x$$

$$\frac{dz}{dx} = 2x+1$$

$$\frac{dv}{dz} = e^z = e^{x^2+x}$$

$$\frac{dv}{dx} = e^z(2x+1) = e^{(x^2+x)}(2x+1)$$

$$\frac{dv}{dx} = (2x+1)e^{x^2+x}$$

Differentiating Using Product Rule

$$\frac{d^2y}{dx^2} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d^2y}{dx^2} = (2x+1) [(2x+1)e^{x^2+x}] + e^{(x^2+x)} \times 2$$

$$\frac{d^2y}{dx^2} = (2x+1) [(2x+1)e^{x^2+x}] + 2e^{x^2+x}$$

But $y = e^{x^2+x}$ \neq $y' = (2x+1)e^{x^2+x}$

$$\frac{d^2y}{dx^2} = y'(2x+1) + 2y$$

Since $\frac{d^2y}{dx^2} = y''$

$$y'' = y'(2x+1) + 2y$$

$$y'(2x+1) + 2y - y'' = 0$$

Let $W_1 = y'(2x+1)$

$$W_2 = 2y$$

$$W_3 = y''$$

For W_1 ,

$$u = y'$$

$$y^{(n)} = y^{(n+1)}$$

$$y^{(n-1)} = y^{(n)}$$

$$v = 2x+1$$

$$v' = 2$$

$$v'' = 0$$

For W_2

$$u = y$$

$$v = 2$$

$$u^n = y^n$$

$$v' = 0$$

For W_3

$$u = y''$$

$$v = 1$$

$$u^n = y^{(n+2)}$$

$$v = 0$$

$$u^{(n-1)} = y^{(n+1)}$$

Recall

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

$$\Rightarrow y^{(n+1)} \cdot (2x+1) + n y^n \times 2 + y^n \times 2 - y^{(n+2)} \times 1 + n y^{(n+1)} \times 0 = 0$$

$$\Rightarrow 2x+1 = y^{(n+1)} + 2n y^n + 2y^n - y^{(n+2)} \neq 0$$

$$y^{(n+2)} = (2x+1) \cdot y^{(n+1)} + 2n y^n + 2y^n$$

$$y^{(n+2)} = (2x+1) \cdot y^{(n+1)} + 2(n+1)y^n$$

Question 2.

$$y = x^3 e^{4x}$$

$$y^n = u^n v + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(n-5)} v^5 + \dots$$

$$u = e^{4x}$$

$$u^n = 4^n e^{4x}$$

$$u^{(n-1)} = 4^{(n-1)} e^{4x}$$

$$u^{(n-2)} = 4^{(n-2)} e^{4x}$$

$$u^{(n-3)} = 4^{(n-3)} e^{4x}$$

$$v = x^3$$

$$v^{(1)} = 3x^2$$

$$v^{(2)} = 6x$$

$$v^{(3)} = 6$$

$$v^{(4)} = 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 3x^2 + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + 5 \left[4^{(5-1)} e^{4x} \cdot 3x^2 \right] + \frac{5(5-1)}{2!} 4^{(5-2)} e^{4x} \cdot 6x + \frac{5(5-1)(5-2)}{3!} 4^{(5-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^{(5)} x^3 e^{4x} + 5 \left[4^{(4)} \cdot 3x^2 e^{4x} \right] + \frac{5(4)}{2!} 4^3 \cdot 6x e^{4x} + \frac{5(4)(3)}{3!} 4^{(2)} e^{4x} \cdot 6$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + \frac{7680}{2!} x e^{4x} + \frac{5760}{3!} e^{4x}$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = e^{4x} [1024 x^3 + 3840 x^2 + 3840 x + 960]$$

$$i) x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

Let

$$W_1 = x^2 y''$$

$$W_2 = x y'$$

$$W_3 = y$$

$$W_1 = x^2 y''$$

$$u = y^2$$

$$V = x^2$$

$$u^n = y^{(n+2)}$$

$$V' = 2x$$

$$u^{(n-1)} = y^{(n+1)}$$

$$V'' = 2$$

$$u^{(n-2)} = y^n$$

$$V''' = 0$$

$$W_2 = x y'$$

$$u = y'$$

$$V = x$$

$$u^n = y^{(n+1)}$$

$$V' = 1$$

$$u^{(n-1)} = y^n$$

$$V'' = 0$$

$$W_3 = y$$

$$u = y$$

$$V = 1$$

$$u^n = y^n$$

$$V' = 0$$

From Leibnitz theorem

$$u^{(n)} V + n u^{(n-1)} V' + \frac{n(n-1)}{2!} u^{(n-2)} V'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V''' + \dots$$

$$\Rightarrow y^{(n+2)} \cdot x^2 + n (y^{(n+1)} \cdot 2x) + \frac{n(n-1)}{2!} y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!} y^{(n-3)} \cdot 0$$

$$\Rightarrow y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{(n+1)} \cdot x + n y^n \cdot 1 + y^n \cdot 1 = 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + \frac{n(n-1)}{2!} 2 y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + [n(n-1) + n + 1] y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$