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17/MHS07/022
Biomedical Engineering
ENG 381 : Engineering Maths 3
Assignment 2

1 If $y = e^{x^2+x}$, show that $y'' = y'(2x+1)+2y$ and hence prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$

$y' = 2x+1(e^{x^2+x})$

$y'' = ?$

Using product rule

$(2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$

Recall $y' = 2x+1(e^{x^2+x})$

$y = e^{x^2+x}$

Substitute y and y' in y''

$y'' = y'(2x+1) + 2y$ QED//

$y'' = y'(2x+1) + 2y$

$A = y''$ $\quad$ $B = y'(2x+1)$ $\quad$ $C = 2y$

$A^n = y^{n+2}$ $\quad$ $u = y'$ $\quad$ $v = 2x+1$ $\quad$ $C^n = 2y^n$

$u^n = y^{n+1}$ $\quad$ $v' = 2$

$B^n = y^{n+1}(2x+1) + ny^n 2$

$A = B+C$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^{n}$ QED //

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(2) Using Leibnitz theorem, $y = x^3 e^{4x}$, determine y^5

Soln

$u = e^{4x}$ $v = x^3$

$u^n = 4^n e^{4x}$ $v' = 3x^2$

$v'' = 6x$

$v''' = 6$

Leibnitz theorem

$y^5 = u^5 v + 5u^4 v' + \frac{5(4)}{2!} u^3 v'' + \frac{5(4)(3)}{3!} u^2 v''' + \frac{5(4)(3)(2)}{4!} u' v^4 + uv^5$

$y^5 = u^5 v + 5u^4 v' + 10u^3 v'' + 10u^2 v''' + 5u' v^4 + uv^5$

Substitute

$y^5 = 4^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0 + 0$

$y^5 = x^3 \cdot 1024e^{4x} + 5(256)e^{4x} \cdot 3x^2 + 10(64e^{4x}) \cdot 6x + 10(16e^{4x}) \cdot 6$

$y^5 = x^3 \cdot 1024e^{4x} + 3840e^{4x} \cdot x^2 + 3840e^{4x} \cdot x + 960e^{4x}$

$y^5 = 1024e^{4x} \cdot x^3 + (x^2 + x)\, 3840e^{4x} + 960e^{4x}$

③ If $\cancel{d}\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$

Soln

$x^2y'' + xy' + y = 0$

$A = x^2y''$

$V = x^2 \qquad u = y''$

$V' = 2x \qquad u^n = y^{n+2}$

$V'' = 2$

$A^n = y^{n+2}\cdot x^2 + ny^{n+1}\cdot 2x + \frac{n(n-1)y^n\cdot \cancel{2}}{\cancel{2}}$

$B = xy'$

$V = x \qquad u = y'$

$V' = 1 \qquad u^n = y^{n+1}$

$B^n = y^{n+1}\cdot x + ny^n$

$C = y$

$C^n = y^n$

$A + B + C = 0$

$y^{n+2}\cdot x^2 + ny^{n+1}\cdot 2x + n(n-1)y^n + y^{n+1}\cdot x + ny^n + y^n = 0$

Rearranging the terms

$y^{n+2}\cdot x^2 + ny^{n+1}\cdot 2x + y^{n+1}\cdot x + n(n-1)y^n + ny^n + y^n = 0$

$x^2\cdot y^{n+2} + (2n+1)xy^{n+1} + n^2y^n - \cancel{ny^n} + \cancel{ny^n} + y^n = 0$

$x^2\cdot y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$ QED