Adebiyi Olumuyiwa
17/Eng06/106
Mathematics Eng 381

(1) If $Y = e^{x^2+x}$

Show that: $y'' = y'(2x+1) + 2y$

$Y = e^{x^2+x}$

$\frac{dy}{dx} = y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = Y'' = (2x+1)(2x+1)e^{x^2+x} + (2)e^{x^2+x}$

Since $y' = (2x+1)e^{x^2+x}$

& $y = e^{x^2+x}$

$Y'' = (2x+1)Y' + 2Y$

$y'' = y'(2x+1) + 2y$

Hence prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Take $y'' = A$, $B = y'(2x+1)$, $C = 2y$

$A' = y'''$

$A^n = y^{(n+2)}$

$B' = y'(2x+1)$, $u = y'$, $u' = y''$, $u'' = y'''$

$v = 2x+1$, $v' = 2$, $v'' = 0$

$B' = y''(2x+1) + n(y')(2)$

$B'' = y'''(2x+1) + n(y'')(2)$

$B^n = y^{n+1}(2x+1) + n(y^n)2$

$C^* = 2y$

$C' = 2y'$

$C^n = 2y^n$

$A^n = B^n + C^n$

$y^{(n+2)} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$y^{(n+2)} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{(n+2)} = y^{n+1}(2x+1) + 2(n+1)y^n$

(2)(i) $Y = x^3e^{4x}$

Using the Leibnitz theorem.

$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \dots$

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$

$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

$u^{iv} = 256e^{4x}$, $u^v = 1024e^{4x}$

$y^5 = (1024e^{4x})(x^3) + (5)(256e^{4x})(3x^2) + \frac{5(4)}{2!}(64e^{4x})(6x) + \frac{5(4)(3)}{3!}(16e^{4x})(6) + \frac{5(4)(3)(2)}{4!}(4e^{4x})(0)$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$

~~$y^5 = 16e^{4x}(64x^3 + 240e^{4x}x^2 + 240e^{4x}x + 60)$~~

$y^5 = 16e^{4x}(64x^3 + 240(x^2+x) + 60)$

(2)(ii)

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' + xy' + y = 0$

take: $A = x^2y''$, $B = xy'$, $C = y$

$A = x^2y''$, $u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^v$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$A' = y'''x^2 + (1)y''2x$

$A'' = y^{iv}x^2 + (2)y'''(2x) + \frac{2(2-1)}{2!}y''(2)$

$A''' = y^vx^2 + 3(y^{iv})(2x) + \frac{3(3-1)}{2!}y'''(2)$

$A^n = y^{(n+2)}x^2 + ny^{(n+1)}(2x) + n(n-1)y^n$

$A^n = y^{(n+2)}x^2 + ny^{(n+1)}(2x) + (n^2-n)y^n$

$B = xy'$, $u = y'$, $u' = y''$, $u'' = y'''$

$v = x$, $v' = 1$, $v'' = 0$

$B' = y''(x) + (1)y'(1) + 0$

$B'' = y'''(x) + (2)y''(1) + 0$

$B^n = y^{n+1}x + ny^{n}$

$C = y$

$C' = y'$

$C^n = y^n$

$A^n + B^n + C^n = 0$

$y^{(n+2)}x^2 + ny^{(n+1)}(2x) + (n^2-n)y^n + y^{n+1}x + ny^{n+1}$

$+ y^n = 0$

$y^{(n+2)}x^2 + ny^{(n+1)}2x + y^{n+1}x + ny^{n+1} + y^n(n^2-n)$

$+ y^n = 0$

$y^{(n+2)}x^2 + y^{(n+1)}(2xn + x + n)$

$A^n + B^n + C^n = 0$

$y^{(n+2)}x^2 + ny^{(n+1)}(2x) + (n^2-n)y^n + y^{n+1}x$

$+ ny^n + y^n = 0$

$y^{(n+2)}x^2 + ny^{(n+1)}(2x) + y^{n+1}x + y^n(n^2-n+n+1) = 0$

$y^{(n+2)}x^2 + ny^{(n+1)}(2x+x) + y^n(n^2+1) = 0$

$y^{(n+2)}x^2 + xy^{(n+1)}(2n+1) + (n^2+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$