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Mechanical Engineering
17/ENG00/011

Assignment 2

1. $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

i. $y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

from the above equation

Part A

$A = y''$, $A'' = y'''$, $A^n = y^{2+n}$

Part B

$B = y'(2x+1)$

$U = y'$, $U^n = y^{n+1}$

$V = 2x+1$

$V' = 2$

$V''' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

Part C,

$C = 2y$

$C^n = 2y^n$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$\therefore y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2 i) $y = x^3 e^{4x}$ $y^{(n)}$.

Let $U = e^{4x}$, $U' = 4e^{4x}$, $U'' = 16e^{4x}$, $U^n = 4^n e^{4x}$

Let $V = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$, $V^{n+1} = 0$

By Leibnitz theorem

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{3!}$$

$$4^{n-3} e^{4x} 6 + c$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 (5) 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} \cdot x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii) $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^n = 0$

for part A.

$A = x^2 y''$

$U = y''$, $U^n = y^{n+2}$

$V = x^2$, $V' = 2x$, $V'' = 2$, $V''' = 0$

$$A^n = (y^{n+2}) x^2 + n (y^{n+1}) \cdot 2x + \frac{n(n-1)}{} \cdot (y^n) 2 + 0$$

$$A^n = x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^n$$

for part B.

$B = xy'$

$U = y'$, $U^n = y^{n+1}$

$V = x$, $V' = 1$, $V'' = 0$

$$B^n = (y^{n+1}) \cdot x + n (y^n) \cdot 1 + 0$$

$$= x y^{(n+1)} + n y^n$$

for part C

$C = y$

$C^n = y^n$

$$\therefore A^n + B^n + C^n = 0$$

$$= x^2 y^{(n+2)} + 2xn y^{(n+1)} + (n^2 - n) y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$= x^2 y^{(n+2)} + x y^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$