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Mechatronics
17/ENG05/038
ENG 381

1. $y = e^{x^2+x}$

Soln

Let $x^2+x = u$

$Y = e^u$

$du/dx = 2x+1$

$dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (2x+1)e^u$
$= (2x+1)e^{x^2+x}$

But $y' = \frac{dy}{dx}$

$y' = (2x+1)e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(2x+1)e^{(x^2+x)}$

Let $u = 2x+1 \Rightarrow du/dx = 2$

$V = e^{x^2+2}$

Differentiating using function of function

Let $z = x^2+2$

$dz/dx = 2x+1$; $dV/dz = e^z$

$\frac{dv}{dx} = e^z + (2x+1) = (2x+1)e^{x^2+2}$

Differentiating using product rule

$\frac{d^2y}{dx^2} = U\,dv/dx + V\,du/dx$

$\frac{d^2y}{dx^2} = (2x+1)[(2x+1)e^{x^2+2}] + 2(e^{x^2+2})$

Recall $Y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+2}$

$y'' = (2x+1)y' + 2y$

$(2x+1)y' + 2y - y'' = 0$

$kl_1 = y'(2x+1)$

$u = y' \quad , \quad u^{(1)} = y'' \quad , \quad u^{(2)} = y''' \quad u^n = y^{n+1}$

$v = 2x+1 \quad , \quad v^{(1)} = 2 \quad , \quad v^{(2)} = 0$

$kl_1^n = u^n v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2} u^{(n-2)} v^{(2)}$

$= y^{n+1}(2x+1) + n(y^n)(2) + 0$

$= (2x+1)y^{n+1} + 2n y^n$

$kl_2 = 2y$

$v = 2 \quad , \quad v^{(1)} = 0$

$u = y' \quad , \quad u^{(1)} = y'' \quad u^n = y^n$

$kl_2^n = u^n v + n u^{(n-1)} v^{(1)}$

$= 2y^n$

$kl_3 = y''$

$u^{(0)} = y'' \quad , \quad u^{(1)} = y^3 \quad , \quad u^n = y^{n+2}$

$v^{(0)} = 1 \quad , \quad v^{(0)} = 0$

$kl_3^n = u^n v + n u^{(n-1)} v^{(1)}$

$= y^{n+2}(1)$

$= y^{n+2}$

$y = kl_1 + kl_2 + kl_3$

$0 = (2x+1)y^{n+1} + 2ny^n + 2y^n - y^{n+2}$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2i $y = x^3 e^{4x}$

$u^{(0)} = e^{4x}$, $u^{(1)} = 4e^{4x}$, $u^{(2)} = 16e^{4x}$, $u^{(3)} = 64e^{4x}$ $u^{(4)} = 256e^{4x}$, $u^n = 4^n e^{4x}$

$v^{(0)} = x^3$, $v^{(1)} = 3x^2$, $v^{(2)} = 6x$, $v^{(3)} = 6$, $v^{(4)} = 0$

$y^n = u^n v^{(0)} + nu^{(n-1)}v^{(1)} + n(n-1)u^{(n-2)}v^{(2)} + \frac{n(n-1)(n-2)}{3\cdot 2}u^{(n-3)}v^{(3)} +$
$\frac{n(n-1)(n-2)(n-3)}{4\cdot 3\cdot 2}u^{(n-4)}v^{(4)}$

$y^n = 4^n e^{4x}(x^3) + n[4^{(n-1)}e^{4x}(3x^2)] + \frac{n(n-1)}{2}[4^{n-2}e^{4x}(6x)] +$
$\frac{n(n-1)(n-2)}{3\cdot 2}[4^{n-3}e^{4x}(6)] + 0$

$y^5 = 4^5 x^3 e^{4x} + 5[4^{(5-1)}e^{4x}(3x^2)] + \frac{5(5-1)}{2}[4^{5-2}e^{4x}(6x)] + \frac{5(5-1)(5-2)}{3\cdot 2}$
$[4^{5-3}e^{4x}(6)]$

$= 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

$= e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$

ii. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$k1_1 = x^2y''$

$v^{(0)} = x^2$, $v^{(1)} = 2x$; $v^{(2)} = 2$ $v^{(3)} = 0$

$u^{(0)} = y^2$, $u^{(1)} = y^3$, $u^{(2)} = y^4$ $u^{(3)} = y^5$, $u^n = y^{n+2}$

$k1_1^n = u^n v^1 + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2}u^{(n-2)}v^{(2)} + \frac{n(n-1)(n-2)}{3\cdot 2}u^{(n-3)}v^{(3)}$

$= y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2}y^n(2) + 0$

$= x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n$

$k1_2 = xy'$

$v = x$, $v^{(1)} = 1$ $v^{(2)} = 0$

$u = y'$, $u^{(1)} = y^2$ $u^{(2)} = y^3$

$k1_2^{(n)} = u^n v + nu^{n-1}v^{(1)} + \frac{n(n-1)}{2}u^{(n-2)}v^{(2)}$

$= y^{n+1}(x) + ny^n(1)$

$$= xy^{n+1} + ny^n$$

$$kl_3 = y$$

$$V = 1 \quad , \quad V^{(1)} = 0$$

$$u = y \quad , \quad u^{(1)} = y^2 \quad , \quad u^n = y^n$$

$$kl_3^n = u^n v^{(0)} + n u^{(n-1)} v^{(1)}$$

$$= y^n$$

$$y = kl_1 + kl_2 + kl_3$$

$$= x^2 y^{n+2} + 2xny^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n$$

$$x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$