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Assignment 2

1) $Y = e^{x^2+x}$

$y^1 = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

2. $y'(2x+1)+2y$

$= (2x+1)e^{x^2+x}(2x+1)+2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1)+2y$

From the above equation:

Part A,

$A = y''$, $A' = y'''$, $A^n = y^{n+2}$

Part B

$B = y'(2x+1)$

Let $y' = U$, $U^n = y^{n+1}$

$V = 2x+1$

$V' = 2$

$V'' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

Part C,

$C = 2y$

$C^n = 2y^n$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$\therefore y^{n+2} = (2x+c)y^{n+1} + 2cn+12y^n$

ii) $y = x^3e^{4x}$, $y^{(n)}$

Let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$

Let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

By Leibnitz Theorem

$$y^n = 4^n e^{4x}x^3 + n \cdot 4^{n-1}e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2}e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3}e^{4x}$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1}e^{4x} + 3n(n-1) \cdot 4^{n-2}e^{4x} + n(n-1)(n-2) \cdot 4^{n-3}e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)\xrightarrow{(4)} \cdot 4^3 e^{4x} \cdot x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024e^{4x} \cdot x^3 + 3840e^{4x} \cdot x^2 + 3840e^{4x} \cdot x + 960e^{4x}$$

$$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

ii) $x^2\frac{d^2y}{dx^2} + xc\frac{dy}{dx} + y = 0$, Show that $x^2y^{(n+2)} + (2n+1)y^{(n+1)} + (n^2+1)y^n = 0$

For part A

$A = x^2y''$

$U = y''$, $U^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$$A^n = (y^{n+2})x^2 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2!} \cdot (y^n) \cdot 2 + 0$$

$$A^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$$

For part B

$B = xy'$

$U = y'$, $U^n = y^{n+1}$

$v = x$, $v' = 1$ $v'' = 0$

$B^n = (y^{n+1}) \cdot x + n(y^n) + 1 + 0$

$= xy^{(n+1)} + ny^n$

For part C,

$C = y$

$C^n = y^n$

$A^n + B^n + C^n = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$