

Assignment 3:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

$$A = x^2 y'' \quad A^n = x^2 y^{n+2} + 2xn y^{n+1} + \frac{2n(n-1)}{2!} y^n$$

$$B = x y' \quad B^n = x y^{n+1} + n y^n$$

$$C^n = y^n \quad C^n = y^n$$

$$A^n + B^n + C^n = 0$$

$$x^2 y^{n+2} + 2xn y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{n+2} + 2x(n-1) y^{n+1} + (n^2 - \cancel{n} + \cancel{n}) y^n = 0$$

$$x^2 y^{n+2} + 2x(n-1) y^{n+1} + (n^2 + 1) y^n = 0$$

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$\begin{aligned} \therefore 2y + (2x+1)y' &= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x}) \\ &= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x} \end{aligned}$$

Therefore

$$y'' = 2y + (2x+1)y'$$

from $y'' = 2y + (2x+1)y'$

$$A = y'' \qquad A^n = y^{n+2}$$

$$B = 2y \qquad B^n = 2y^n$$

$$C = (2x+1)y' \qquad C^n = (2x+1)y^{n+1} + 2ny^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+1} = (2x+1)y^n + 2(n+1)y^{n-1}$$

Assignment 2

$$y = x^3 e^{4x}$$

$$y^n = u^n v + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$$

$$y^5 = u^5 v + 5u^4 v^2 + 10u^3 v^3 + 10u^2 v^4 + 5u v^5 + u v^5$$

$$u = e^{4x}$$

$$v = x^3$$

$$\begin{aligned} y^5 &= 4^5 e^{20x} \cdot x^3 + (5)(4^4) e^{16x} \cdot (3x^2) + 10(4^3 e^{12x})(6x) + 10(4^2 e^{8x})(6) + 0 \\ &= 1024x^3 e^{20x} + 3,840x^2 e^{16x} + 3,840x e^{12x} + 960 e^{8x} \\ &= 64e^{8x} (16x^3 + 240x^2 + 240x + 15) \end{aligned}$$