AGWANIRU ROSEMARY
17/ENG/011003
CHEMICAL ENGINEERING

1) $y = e^{x^2+x}$
Differentiating
$y = y'$ and $y = y''$
$y' = (2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$
$y'' = (2x+1)y' + y \cdot 2$
Therefore
$y'' = y'(2x+1) + 2y$
Finding the nth derivation of $y''$
$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^n \cdot 2 + 2y^n$
$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

2) $y = x^3 e^{4x}$ determine $y^{(5)}$
Using leibnitz rule
$y^n = u^n v + n(u^{(n-1)})v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-1)(n-2)u^{(n-3)}}{3!}$
$v''' + \frac{n(n-1)(n-2)(n-3)}{4!}u^{(n-4)}v^{(4)} + \dots$

Taking
$U = e^{4x}$ $V = x^3$
$u^n = 4^n e^{4x}$ $v^{(1)} = 3x^2$
$u^{(n-1)} = 4^{(n-1)}e^{4x}$ $v^{(2)} = 6x$
$u^{(n-2)} = 4^{(n-2)} \cdot e^{4x}$ $v^{(3)} = 6$
$u^{(n-3)} = 4^{(n-3)} \cdot e^{4x}$ $v^{(4)} = 0$
$u^{(n-4)} = 4^{(n-4)}e^{4x}$ $v^{(5)} = 0$
$u^{(n-5)} = 4^{(n-5)} \cdot e^{4x}$

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$y^{n} = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} 6x \cdot 4^{(n-2)} e^{4x} +$

$\frac{n(n-1)(n-2)}{3!} \cdot 6 \cdot 4^{(n-3)} e^{4x} + 0$

$y^{(n)} = x^3 4^n e^{4x} + 3n x^2 + 4^{(n-1)} e^{4x} + \frac{n(n-1)}{2} 6x \cdot 4^{(n-2)} e^{4x} +$

$\frac{6(n-1)(n-2)}{6} \cdot 4^{(n-3)} e^{4x} + 0$

$\cancel{y^{(n)} = x^3 4^{(n)} e^{4x}}$

$y^{(n)} = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + (n-1)(n-2)$
$4^{(n-3)} e^{4x} + 0$

$y^n = y^5$ i.e $n = 5$

$y^5 = x^3 4^{(5)} e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)(5-1) x \cdot 4^{(5-2)} \cdot e^{4x} +$
$5(5-1)(5-2) 4^{(5-3)} e^{4x}$

$y^{(5)} = 1024 x^3 + 3340 x^2 e^{4x} + 3340 x^{4x} + 960 e^{4x}$

(3) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

show that

$x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Solution:

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y^{(0)} = 0$

$u_1 = x^2 y''$

$u = y''$

$u^n = y^{(n+2)}$ $\qquad v = x^2$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v^{(1)} = 2x$

$u^{(n-2)} = y^n$ $\qquad v^{(2)} = 2$

$v^{(3)} = 0$

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$w_2 - xy'$

$u = y'$ $\qquad v = x$

$u^n = y^{(n+1)}$ $\qquad v^{(1)} = 1$

$u^{(n-1)} = y^n$ $\qquad v^{(2)} = 0$

$w_2 = y^{(0)}$

$u = y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v^{(1)} = 0$

$$y^{(n)} = u^n v + n\,u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \ldots$$

$$w_1^{(n)} = y^{(n+2)} \cdot x^2 + n\left(y^{(n+1)} \cdot 2x\right] + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + 0$$

$$w_2^{(n)} = y^{(n+1)} \cdot x + n\left[y^n \cdot 1\right] + 0$$

$$w_3^{(n)} = y^n$$

$$y^{(n+2)} x^2 + 2nx\,y^{(n+1)} + \frac{n(n-1)}{2} \cdot 2y^n + y^{(n+1)} \cdot x + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2nx \cdot y^{(n+1)} + n(n-1) y^n + y^{(n+1)} \cdot x + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2nx\,y^{(n+1)} + xy^{(n+1)} + n(n-1) y^n + ny^n + y^n = 0$$

$$\cancel{x^2 y^{(n+3)}}$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1) y^n = 0$$