ENG 381

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elect/elect.

Assignment II

(1) if $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$

Hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$\Rightarrow$ $y = e^{x^2+x}$

$y = e^{x^2} \cdot e^x$ (Applying Product Rule) $u\frac{dv}{dx} + v\frac{du}{dx}$

$u = e^{x^2}$, $v = e^x$ $\frac{dv}{dx} = e^x$, $\frac{du}{dx} = 2xe^{x^2}$

$y^{(1)} = e^x(2xe^{x^2}) + e^{x^2} \cdot e^x$

$y^{(1)} = e^{x^2+x}(2x) + e^{x^2+x}$ $\Rightarrow$ $y^{(1)} = e^{x^2+x}(2x+1)$

Applying Product Rule

$u = e^{x^2+x}$ $v = 2x+1$

$\frac{du}{dx}$ $e^{x^2+x}(2x+1)$ $\frac{dv}{dx} = 2$

$\therefore y'' = 2(e^{x^2+x}) + (2x+1)(2x+1)e^{x^2+x}$

Recall $y = e^{x^2+x}$ $\$$ $y^{(1)} = e^{x^2+x}(2x+1)$

$\therefore$ $y'' = y'(2x+1) + 2y$

$\Rightarrow$ for $y'(2x+1)$

$u = y'$, $v = 2x+1$

$u^n = y^{(n+1)}$ $v' = 2$

$u^{n-1} = y^n$ $v'' = 0$

for $2y$

$u = 2y$ $v = 1$

$u^n = 2y^n$ $v' = 0$

using Leibnitz theorem.

$\therefore y^{n+2} = y^{(n+1)} \cdot 2x+1 + ny^n \cdot 2 + 2y^n \cdot 1$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) Using the leibnitz Theorem given that

i) $y = x^3 e^{4x}$; determine $y^{(5)}$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, ~~Show that~~

show that $x^2 y^{(n+2)} + (2n+1)\, x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Soln

1) $y = x^3 e^{4x}$

$u = e^{4x}$, $V = x^3$

$u^n = 4^n e^{4x}$ $\qquad V^{(1)} = 3x^2$

$u^{n-1} = 4^{n-1} e^{4x}$ $\qquad V^{(2)} = 6x$

$u^{n-2} = 4^{n-2} e^{4x}$ $\qquad V^{(3)} = 6$

$u^{n-3} = 4^{n-3} e^{4x}$ $\qquad V^{(4)} = 0$

when $n = 5$

$\therefore y^5 = 4^5 e^{4x}$

$u^{(5-1)} = 4^4 e^{4x}$

$u^{(5-2)} = 4^3 e^{4x}$

$u^{(5-3)} = 4^2 e^{4x}$

$u^{(5-4)} = 4^1 e^{4x}$

Using leibnitz theorem.

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{n-3} \times v^3$

$+ - - -$

Hence : $y^{(5)} = u^{(5)} v + 5u^{(4)} v' + 10 \cdot 4 u^{(3)} V^2 + 10 \quad u^2 v^3$

$+ \ldots \quad 5 u' v^4 + - - -$

$\therefore y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 \cdot 3x^2 + 10 \cdot 4^3 \cdot e^{4x} + 10 \cdot 4^2 \cdot e^{4x} + \cancel{5 \cdot 4' \cdot 0}$

$y^{(5)} = 1024 e^{4x} \cdot x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} 6x + 160 e^{x} \cdot 6 + 0$

$y^{(5)} = 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} \cdot 6x + 960 e^{4x}$

$y^{(5)} = e^{4x}(1024x^3 + 38403x^2 + 3840x + 960)$

ii) $\Rightarrow x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\Rightarrow x^2 y^{(2)} + x y^{(1)} + y = 0$

Applying leibnitz theorem.

$\Rightarrow w_1 = x^2 y''$

$u = y^{(2)}$. , $v = x^2$

$u^n = y^{n+2}$ $v' = 2x$

$u^{n+1} = y^{n+1}$ $v'' = 2$

$u^{n-2} = y^n$ $v''' = 0$

$\Rightarrow w_2 = x y^{(1)}$

$u = y'$ $v = x$

$u^n = y^{n+1}$ $v' = \cancel{0}\, 1$

$u^{n-1} = y^n$ $v'' = 0$

$\Rightarrow w_3 = y$

$u = y$ $v = 1$

$u^n = y^n$ $v' = 0$

$\Rightarrow \therefore w_1 + w_2 + w_3 = 0$

using:

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \cdots$$

$$y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot 2x + \frac{n(n-1)}{\cancel{2}!} \cdot y^n \cdot \cancel{2} + y^{(n+1)} \cdot x + n x y^n \cdot 1 + y^n \cdot 1 = 0$$

$$x^2 y^{(n+2)} + 2xn \cdot y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n^2 - \cancel{n} + \cancel{n} + 1) = 0$$

$$\therefore x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n^2+1) = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$