$= y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$= y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

Then

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

$= y^{(n+1)}(2x+1) + 2(n+1)y^{n}$

Therefore

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2(n+1)y^{n}$ 4.

2) Using the Leibnitz theorem, given that

i) $y = x^3 e^{4x}$, determine $y^{(n)}$

ii) $x^2\frac{d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$

Show that

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

i) $y = x^3e^{4x}$

$u = e^{4x}$ $\quad v = x^3$

$u^{(1)} = 4e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(2)} = 16e^{4x}$ $\quad v^{(2)} = 6x$

$u^{(n)} = 4^n e^{4x}$ $\quad v^{(3)} = 6$

$u^{(n-1)} = 4^{(n-1)}e^{4x}$ $\quad v^{(4)} = 0$

$u^{(n-2)} = 4^{(n-2)}e^{4x}$

$u^{(n-3)} = 4^{(n-3)}e^{4x}$

$\therefore 4^n e^{4x}\cdot x^3 + n\cdot 4^{(n-1)}e^{4x}\cdot 3x^2 + \frac{n(n-1)}{2!}\cdot 4^{(n-2)}e^{4x}\cdot 6x +$

$\frac{n(n-1)(n-2)}{\cancel{1\times2\times3}}\cdot 4^{(n-3)}e^{4x}\cdot 6 + \dots 0$

$= 4^n x^3 e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + 3n(n-1)x 4^{(n-2)}e^{4x} +$

$= n(n-1)(n-2)4^{(n-3)}e^{4x}$

Recall $y^{(n)} = y^{(5)}$

$y^{(5)} = 4^5 x^3 e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x} + 3(5)(5-1)x 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$

$y^{(5)} = 4^5 x^3 e^{4x} + 15x^2 4^4 e^{4x} + 60x 4^3 e^{4x} + 60\cdot 4^2 e^{4x}$

$= 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$

$\therefore y^{(5)}$ of $x^3e^{4x} = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

(ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

Differentiating separately, n times

x^2y''		xy'		$y^{(n)}$
$u = y''$	$v = x^2$	$u = y'$	$v = x$	
$u^{(n)} = y^{(n+2)}$	$v' = 2x$	$u^{(n)} = y^{(n+1)}$	$v' = 1$	
$u^{(n-1)} = y^{(n+1)}$	$v'' = 2$	$u^{(n-1)} = y^{(n)}$	$v'' = 0$	
$u^{(n-2)} = y^{(n)}$	$v''' = 0$			

for x^2y''

$= y^{(n+2)}\cdot x^2 + n\cdot y^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}\cdot y^{(n)}\cdot \cancel{2}$

$= x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$

for xy'

$= y^{(n+1)}\cdot x + n\cdot y^{(n)}\cdot 1 \quad = xy^{(n+1)} + ny^{(n)}$

for y

$= y^{(n)}$

Adding them,

$= x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$= x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + [n(n-1) + n + 1]y^{(n)} = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2 - \cancel{n} + \cancel{n} + 1)y^{(n)} = 0$

$= x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

ELUWA TOCHUKWU DIVINE
17/ENG07/010
Petroleum Engineering
ENG 381 Assignment 2.

1) If $y = e^{x^2+x}$
show that $y'' = y'(2x+1) + 2y$
and hence prove that
$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Answer

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$ ∴ differentiating $x^2+x = 2x+1$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

Rewriting

$y'' = (2x+1)y' + 2y$

$\therefore y'' = y'(2x+1) + 2y$

* To find nth term,

lets say $P = y'(2x+1)$

$Q = 2y$

P

$u = y'$ $\quad v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $\quad v^{(1)} = 2$

$u^n = y^{(n+1)}$ $\quad v^{(2)} = 0$

Q

$u = y$ $\quad v = 2$

$u^{(n)} = y^{(n)}$ $\quad v^{(1)} = 0$

for P

$= y^{(n+1)}(2x+1) + ny^{(n+1-1)} \cdot 2 + \frac{n(n-1)}{1 \times 2} y^{(n+1-2)} \cdot 0$

$\therefore y^{(n+1)}(2x+1) + ny^{(n)}(2)$

For Q

$\therefore y^{(n)} \cdot 2 + ny^{(n-1)} \cdot 0$

$= y^{(n)} \cdot 2$

Adding P and Q

$= y^{(n+1)}(2x+1) + ny^{(n)}(2) + y^{(n)} \cdot 2$