OHWO JAMES OTAOGHENE
17/ENG03/039
CIVIL ENGINEERING

ENG 381 ENG MATHEMATICS

1a $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{\delta}{\delta x}(e^{x^2+x}) + e^{x^2+x}\frac{\partial}{\partial x}(2x+1)$ (Product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proved

1b $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $u' = y'''$ hence $u^n = y^{n+2}$

$v = 1$ $v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

$w_1' = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$ —— (1)

$w_2 = y'(2x+1)$

$u = y'$ $u' = y''$ $u'' = y'''$ Hence $u^n = y^{n+1}$

$V = 2x+1$ $V' = 2$ $V'' = 0$

$w_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 +$

$W_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \cdots$

$W_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$W_2 = y^{n+1}(2x+1) + 2ny^n$ --- (ii)

W_3

$U = y \quad U' = y^1$ Hence $U^n = y^n$

$V = 2 \quad V' = 0$

$W_3 = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!} U^{n-2} V^2 +$

$W_3 = y^n(2) + ny^{n-1}(0) +$

$W_3 = 2y^n$ ----- (iii)

Putting back into the degenerate eqn

$W_1 - W_2 - W_3 = 0$

$y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \Rightarrow$ Proven

Scanned by CamScanner

Question 2

2a $y = x^3 e^{4x}$

Using Leibnitz theorem

$U = e^{4x}$ $U' = 4e^{4x}$ $U'' = 16e^{4x}$ $U''' = 64e^{4x}$ $U^{iv} = 256$

$V = x^3$ $V' = 3x^2$ $V'' = 6x$ $V''' = 6$ $V^{iv} = 0$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3} V^2$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} U^{n-4} V^2 + \ldots$$

$$y^n = 4^n e^{4x}(x^3) + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-3} e^{4x}(6x$$

$$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x}(6)$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4}(0)^2$$

$4^{n-3} e^{4x}$

$y^n = 4^n x^3 e^{4x} + n 3x^2 4^{n-1} e^{4x} + 3x\, n(n-1) 4^{n-2} + n(n-1)(n-2)$

$y^n = e^{4x} 4^{n-3}[x^3 4^3 + 3n x^2 4^2 + 3x n(n-1) 4 + n(n-1)(n-2)]$

$y^n = e^{4x} 4^{n-3}[64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2)]$

$y^5 = e^{4x} 4^{5-3}[64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2)]$

$y^5 = e^{4x} 4^2 [64x^3 + 240x^2 + 240x + 36]$

$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$

2b $x^2 \frac{d^2y}{dx^2} + \frac{n\,dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$W_1 + W_2 + W_3 = 0$

$W_1 = x^2 y''$

$U = y''$ $U' = y'''$ $U'' = y^{iv}$ $U''' = y^v$ Hence $U^n = y^{n+2}$

$V = x^2$ $V' = 2x$ $V'' = 2$ $V''' = 0$

$$W_1 = U^n(V) + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2}(V'') + \frac{n(n-1)(n-2)}{3!}$$

$U^{n-3} V''' + \ldots$

$W_1 = y^{n+2}(x^2) + n y^{n+1}(2x) + n(n-1) y^n(2) + 0$

$W_1 = x^2 y^{n+2} + 2xn\, y^{n+1} + y^n n(n-1)$.

Scanned by CamScanner

$w_2 = xy'$

$u = y'$ $u' = y''$ $u'' = y'''$ Hence $u^n = y^{n+1}$

$v = x$ $v' = 1$ $v'' = 0$

$w_2 = u^n(v) + n\,u^{n-1}(v') + \frac{n(n-1)}{2!}\,u^{n-2}\,v^2$

$w_2 = y^{n+1}(x) + n y^n (1) + 0$

$w_2 = xy^{n+1} + ny^n$

$w_3 = y$

$u = y$ $u' = y'$ Hence $u^n = y^n$

$v_1 = 1$ $v' = 0$

$w^3 = u^n(v) + n\,u^{n-1}(v')$

$= y(1) + 0 = y$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + (n+1))$

$x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$