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17/ENG07/019
Petroleum Engineering
ENG 381
Assignment II

1. If $y = e^{x^2+x}$
Show that $y'' = y'(2x+1) + 2y$ and hence, prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y = e^{x^2+x}$
Differentiating.
$y' = (2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$

$\therefore\ y'' = (2x+1)y' + 2y$

finding the nth term.
Assuming that $P_1 = (2x+1)y'$
$P_2 = 2y$

for P_1		for P_2	
$u = y'$	$v = 2x+1$	$u = y$	$v = 2$
$u^n = y^{(n+1)}$	$v' = 2$	$u^n = y^n$	$v' = 0$
$u^{(n-1)} = y^n$	$v'' = 0$		

for P_1
$= y^{(n+1)}(2x+1) + ny^n(2)$
for P_2
$= y^n(2)$
$= y^{(n+1)}(2x+1) + ny^{(n)}2 + y^n(2)$
$= y^{(n+1)}(2x+1) + 2ny^n + 2y^{(n)}$
$= y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$
$\therefore\ y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1)$
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2. Using the Leibnitz theorem, given that

(a) $y = x^3 e^{4x}$, determine $y^{(5)}$

$$y^n = \sum_{r=0} u^{(n-r)} v^{(r)}$$

Soln

$u = e^{4x}$ $\quad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\quad$ $v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad$ $v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad$ $v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad$ $v'''' = 0$

$u^{(n-4)} = 4^{(n-4)} e^{4x}$ $\quad$ $v''''' = 0$

$u^{(n-5)} = 4^{(n-5)} e^{4x}$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)} \ldots$$

Substituting into the general Solution

$$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{(n-3)} e^{4x} \cdot 6 + \cancel{\frac{n(n-1)(n-2)(n-3)}{4!}} \cdot 4^{(n-4)} e^{4x} \cdot 0 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \cdot 4^{(n-5)} e^{4x} \cdot 0$$

$$\therefore y^n = x^3 4^n e^{4x} + 3n x^2 4^{(n-1)} e^{4x} + \frac{n(n-1) 6x}{2!} \cdot 4^{(n-2)} e^{4x} + \frac{6n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} + 0 + 0$$

$$y^n = x^3 4^n e^{4x} + 3n x^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$$

Recall $y^n = y^5 = n = 5$

$$y^{(5)} = x^3 4^5 e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)x(5-1) 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$$

$$= [illegible] x^3 e^{4x} + 15(256) x^2 e^{4x} + 15(256) x e^{4x} + 60(16) e^{4x}$$

$$y^{(5)} = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

2ii) $\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Soln

1^{st} term Differentiate x^2y'' n times

$V = x^2 \qquad u = y''$

$V' = 2x \qquad u^n = y^{(n+2)}$

$V'' = 2$

$= y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)y^n(2)}{2!}$

2^{nd} term Diff xy' n times

$V = x \qquad u = y'$

$V' = 1 \qquad u^n = y^{(n+1)}$

$y^{(n+1)}x + ny^n$

3^{rd} term Differentiate y n times

$u = y$

$u^n = y^n$

Adding the terms.

$y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)y^n(\cancel{2})}{\cancel{2}!} + y^{(n+1)}x + ny^n + y^n = 0$

$x^2y^{(n+2)} + 2xny^{(n+1)} + ny^n(n-1) + xy^{(n+1)} + ny^n + y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + n(n-1)y^n + ny^n + y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n[n^2-n+n+1] = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$