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17/ENG03/014
CIVIL ENGINEERING
ENG 381 ENGINEERING MATHEMATICS
QUESTION 1

1a) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x} \frac{d}{dx}(2x+1)$ (Product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proven

1b) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1,

$u = y''$ $u' = y'''$ ∴ hence $u^n = y^{n+2}$

$v = 1$ $v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$

$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$ ...... (1)

w_2,

$w_2 = y'(2x+1)$

$u = y'$ $u' = y''$ $u'' = y'''$ Hence $u^n = y^{n+1}$

$V = 2x+1$ $V' = 2$ $V'' = 0$

$w_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$

$w_2 = y^{n+1}(2x+1) + n y^n (2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \dots$

$w_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$w_2 = y^{n+1}(2x+1) + 2ny^n$ (ii)

w_3

$u = y \quad u' = y'$ Hence $u^n = y^n$

$v = 2, \quad v' = 0$

$w_3 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \dots$

$w_3 = y^n(2) + ny^{n-1}(0) + \dots$

$w_3 = 2y^n$ (iii)

Putting back into the degenerate eqn

$w_1 - w_2 - w_3 = 0$

$y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$ => Proved

QUESTION 2

2a.) $y = x^3 e^{4x}$

Using Leibnitz theorem

$U = e^{4x}$ $U^{I} = 4e^{4x}$ $U^{II} = 16e^{4x}$ $U^{III} = 64e^{4x}$

$U^{IV} = 256 e^{4x}$

$V = x^3$ $V^{I} = 3x^2$ $V^{II} = 6x$ $V^{III} = 6$ $V^{IV} = 0$

Hence $U^n = 4^n e^{4x}$

$y^n = U^n V + nU^{n-1} V^1 + \frac{n(n-1)}{2!} U^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3} V^2$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} U^{n-4} V^2 + \dots$

$y^n = 4^n e^{4x}(x^3) + n4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-3} e^{4x}(6x)$

$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)^2$

$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1} e^{4x} + 3x n(n-1) 4^{n-3} + n(n-1)(n-2)$
$4^{n-3} e^{4x} + 0$

$y^n = e^{4n} 4^{n-3} (x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2)]$

$y^n = e^{4x} 4^{n-3} [64x^3 + 48x^2 n + 12xn(n-1) + n(n-1)(n-2)]$

$y^5 = e^{4x} 4^{5-3} [64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2)]$

$y^5 = e^{4x} 4^2 [64x^3 + 240x^2 + 240x + 36]$

$y^5 = 16 e^{4x} (64x^3 + 240x^2 + 240x + 36)$

2b.) $x^2 \frac{d^2y}{dx^2} + n\frac{dy}{dx} + y = 0$

$x^2 y^{x} + xy^1 + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2 y^{II}$

$U = y^{II}$ $U^1 = y^{III}$ $U^{II} = y^{IV}$ $U^{III} = y^{V}$ Hence $U^n = y^{n+2}$

$V = x^2$ $V^1 = 2x$ $V^{II} = 2$ $V^{III} = 0$

$w_1 = U^n(V) + nU^{n-1}V^1 + \frac{n(n-1)}{2!} U^{n-2}(V^{II}) + \frac{n(n-1)(n-2)}{3!}$
$U^{n-3} V^{III} + \dots$

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$w_1 = y^{n+2}(x^2) + n y^{n+1}(2x) + \frac{n(n-1)}{2!} y^n (2) + 0$

$w_1 = x^2 y^{n+2} + 2xn y^{n+1} + y^n n(n-1)$

$w_2 = xy'$

$u = y'$ $u' = y''$ $u'' = y'''$ Hence $u^n = y^{n+1}$

$v = x$ $v' = 1$ $v'' = 0$

$w_2 = u^n(v) + n u^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v''$

$w_2 = y^{n+1}(x) + n y^n (1) + 0$

$w_2 = x y^{n+1} + n y^n$

$w_3 = y$

$u = y$ $u' = y'$ Hence $u^n = y^n$

$v = 1$ $v' = 0$

$w_3 = u^n(v) + n u^{n-1}(v')$

$= y(1) + 0 = y$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(x^2) + n y^{n+1}(2x) + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n (n(n-1) + (n+1))$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2+1) = 0$