

Solution

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$1. \quad y''(2x+1) + 2y$$

$$= (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$$

$$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

$$\text{but } y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'' = y''(2x+1) + 2y$$

from the above equation

Part A,

$$A = y'', A' = y''', A'' = y^{(4)}$$

Part B

$$B = y'(2x+1)$$

$$U = y', U' = y^{n+1}$$

$$V = 2x+1$$

$$V' = 2$$

$$V'' = 0$$

$$\therefore B'' = (y^{n+1})(2x+1) + n(y^n)(2) + 0$$

$$B'' = (2x+1)y^{n+1} + 2ny^n$$

Part C,

$$C = 2y$$

$$C'' = 2y^n$$

$$A'' = B'' + C''$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$$

$$\therefore y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$$

$$2a) y = x^3 e^{4x}, y^{(5)}$$

$$\text{Let } u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u''' = 64e^{4x}$$

$$\text{Let } v = x^3, v' = 3x^2, v'' = 6x, v''' = 6, v^{(4)} = 0$$

By Leibniz theorem

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n+1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^n e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n+1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

$$11) \frac{x^2 d^2 y}{dx^2} + 2x \frac{dy}{dx} + y = 0, \text{ Show that } x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$$

for part A

$$A = x^2 y''$$

$$u = y'', u^n = y^{n+2}$$

$$v = x^2, v' = 2x, v'' = 2, v''' = 0$$

$$A^n = (y^{n+2}) x^2 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2!} \cdot (y^n) \cdot 2 + 0$$

$$A^n = x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n+1) y^n$$

for Part B

$$B = x y'$$

$$u = y', u^n = y^{n+1}$$

$$v = x, v' = 1, v'' = 0$$

$$B^n = (y^{n+1}) x + n(y^n) \cdot 1 + 0$$

$$= x y^{(n+1)} + n y^n$$

For part (

$$C = y$$

$$C^n = y^n$$

$$A^n + B^n + C^n = 0$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$$

$$= x^2 y^{(n+2)} + 2xy^{(n+1)}(n+1) + y^n(n^2 - n + n + 1) = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$