II.) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that:

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

for Part A

$A = x^2y''$

$U = y'', U^n = y^{n+2}$

$V = x^2, V' = 2x, V'' = 2, V''' = 0$

$A^n = (y^{n+2})x^2 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2!} \cdot (y^n)2 + 0$

$A^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

part B

$B = xy$

$U = y', U^n = y^{n+1}$

$V = x, V' = 1, V'' = 0$

$B^n = (y^{n+1}) \cdot x + n(y^n) \cdot 1 + 0$

$= xy^{(n+1)} + ny^n$

for Part C

$C = y$

$C^n = y^n$

$A^n + B^n + C^n = 0$

$= x^2y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$x^2y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^2 = 0$

ENG 381
Engineering Mathematics
Assignment II

1) $y'(2x+1)+2y$
$=(2x+1)e^{x^2+x}(2x+1)+2(e^{x^2+x})$
$=(2x+1)^2 e^{x^2+x}+(2x+1)e^{x^2+x}$
but $y''=2e^{x^2+x}+(2x+1)e^{x^2+x}$
$\therefore y''=y'(2x+1)+2y$
from the above equation
Part A, $A=y''$, $A'=A'''$, $A^n=y^{2+n}$
Part B, $B=y'(2x+1)$
$U=y'$, $U'=y^{n+1}$
$V=2x+1$, $V'=2$, $V''=0$
$\therefore B^n=(y^{n+1})(2x+1)+n(y^n)(2)+0$, $B_n=(2x+1)y^{n+1}+2ny^0$
Part C, $C=2y$
$C^n=2y^n$
$\therefore A^n=B^n+C^n$, $y^{n+2}=(2x+1)y^{n+1}+2ny^n+2y^n$
$y^{n+2}=(2x+1)y^{(n+1)}+2(n+1)y^n$

2) $y=x^3e^{4x}$ determine y^5
Let $U=e^{4x}$, $U'=4e^{4x}$, $U''=16e^{4x}$, $U^n=4^ne^{4x}$
Let $V=x^3$, $V'=3x^2$, $V''=6x$, $V'''=6$, $V^{IV}=0$
by Leibnitz theorem
$y^n=4^ne^{4x}\cdot x^3+n\cdot 4^{n-1}e^{4x}\cdot 3x^2+\frac{n(n-1)}{2!}4^{n-2}e^{4x}6x+\frac{n(n-1)(n-2)}{3!}\cdot 4^{n-3}e^{4x}\cdot 6+C$

$y^n\cdot 4^ne^{4x}\cdot x^3+3x^2n\cdot 4^{n-1}e^{4x}+3n(n-1)\cdot 4^{n-2}e^{4x}x+n(n-1)(n-2)\cdot 4^{n-3}e^{4x}$
$y^5=4^5e^{4x}\cdot x^3+3x^2(5)\cdot 4^4e^{4x}+3(5)(4)\cdot 4^3e^{4x}\cdot x+(5)(4)(3)\cdot 4^2e^{4x}$
$y^5=64e^{4x}(16x^3+60x^2+60x+15)$