James Evidence
17/ENG06/049
Mechanical Engineering
Assignment 2
6-10-19

Solution

1 $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y''' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} . (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

from the above equation

Part A.

$A = y''$, $A' = y'''$, $A^n = y^{n+n}$

part B

$B = y'(2x+1)$

$u = y'$, $u'' = y^{x+1}$

$v = 2x+1$

$v' = 2$

$v''' = 0$

$\therefore B^n = (y^{x+1})(2x+1) + 2n(y^n)(2) + 0$

$B^n = (2x+1)y^{x+1} + 2ny^n$

Part C

$C = 2y$

$C^n = 2y^n$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

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$\therefore y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2. Solution

(i) $y = x^3 e^{4x}, y^5$

let $u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u^n = 4^n e^{4x}$

let $v = x^3, v' = 3x^2, v'' = 6x, v''' = 6, v'''' = 0$

By liebniz theorem

$\frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 + 0$

$y^n = 4^n e^{4x} \cdot x^4 + 3x^2 n 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2)$
$4^{n-3} e^{4x}$

$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} x + (5)(4)(3)$
$4^2 e^{4x}$

$y^5 = 1024e^{4x} \cdot x^2 + 3840e^{4x} \cdot x^2 + 3840e^{4x} \cdot x + 960e^{4x}$

$y^5 = 64e^{4x}(16x^2 + 60x^2 + 60x + 15)$

b] $x^3 \frac{d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$

For Part A

$A = x^2 y''$

$u = y'', u^n = y^{n+2}$

$v = x^2, v' = 2x, v'' = 2, v''' = 0$

$A^n = (y^{n+3})x^3 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2!}(y^n) \cdot 2 + 0$

$A^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y''$

For Part B

$A = xy'$

$u = y', u^n = y^{n+1}$

$v = x, v' = 1, v'' = 0$

$B'' = y^{(n+1)}x + n(y^n) \cdot 1 + 0$

$= xy^{(n+1)} + ny^n$

for part c

$C = y$

$C^n = y^n$

$\therefore A^n + B^n + C^n = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2 y^{(n+2)} + x^{n+1}(2n+1) + y^n(n^2 - n + n + q) = 0$

$= x^2 y^{(n+2)} + 2n+1)xy^{(n+1)} + (n^2 + q)y^n = 0$ //