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1 $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x}\frac{d}{dx}(2x+1)$ (product rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \rightarrow$

1b). $y'' - y'(2x+1) - 2y = 0$

using Leibnitz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

Degenerate eqn: $w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $u' = y'''$ hence $u^n = y^{n+2}$

$V = 1$ $V' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$

$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

W_2

$W_2 = u'(2x+1)$

$u = y'$ $u' = y''$ $u'' = y'''$, hence $u^n = y^{n+1}$

$V = 2x+1$ $V' = 2$ $V'' = 0$

$W_2 = u_n V + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} V^3 + \dots$

$w_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!} y^{n-1}(0) + \cdots$

$w_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$w_2 = y^{n+1}(2x+1) + 2ny^n - - - - - (2)$

w^3

$u = y \qquad u' = y' \qquad \text{Hence } u^n = y^n$

$v = 2 \qquad v' = 0$

$w^3 = u^n v + nu^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + - - - -$

$w^3 = y^n(2) + ny^{n-1}(0) + - - - - -$

$w^3 = \underline{2y^n} - - - - - (3)$

Putting back into the degenerate eqn.

$w_1 - w_2 - w_3 = 0$

$y^{n+2} - (y^{n+1}(2n+1) + 2ny^n) - 2y^n = 0.$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1).$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1).$

Question 2

$y = x^3 e^{4x}$

Using leibnitz theorem.

$u = e^{4x}, \; u' = 4e^{4x}, \; u'' = 16e^{4x}, \; u''' = 64e^{4x}, \; u^{iv} = 256e^{4x}$

$v = x^3 \qquad v' = 3x^2 \qquad v'' = 6x \qquad v''' = 6 \qquad v^{iv} = 0$

Hence $u^n = 4^n e^{4x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1) u^{n-2}}{2!} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^{iv} + \ldots$$

$$y^n = 4^n e^{4x}(x^3) + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1) 4^{n-2}}{2!} e^{4x}(6x) + \frac{n(n-1)(n-2)}{3!}$$

$$4^{n-3} e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)$$

$$y^n = 4^n x^3 e^{4x} + n 3x^2 4^{n-1} e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3} e^{4x} + 0$$

$$y^n = e^{4x} 4^{n-3} (x^3 4^3 + 3n x^2 \cdot 4^2 + 3xn(n-1)4 + n(n-1)(n-2)]$$

$$y^n = e^{4x} 4^{n-3} (64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2)] = (y^5)$$

$$y^5 = e^{4x} 4^{5-3} (64x^3 + 48x^2(5) + (12x \cdot 5(5-1)) + 5(5-1)(5-2)]$$

$$y^5 = e^{4x} 4^2 (64x^3 + 240x^2 + 240x + 36)$$

$$y^5 = 16 e^{4x} (64x^3 + 240x^2 + 240x + 36)$$

2b) $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2 y''$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^{v}$ Hence, $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$ $v'' = 2$, $v''' = 0$

w_2

$$w_1 = U^n(v) + nU^{n-1}v' + \frac{n(n-1)}{2!}U^{n-2}(v'') + \frac{n(n-1)(n-2)}{3!}u^{n-3}v'''$$

$$w_1 = y^{n+2}(x^2) + n(y^{n+1})(2x) + \frac{n(n-1)}{2}y(2) + 0$$

$$w_1 = x^2y^{n+2} + 2xn\underline{y}^{n+1} + y_n(n-1)$$

$\underline{w^2 = ny'}$

$u = y'$, $u' = y''$, $u'' = y'''$ Hence $U^n = y^{n+1}$

$V = x$ $V_1' = 1$ $V'' = 0$.

$$W_2 = U^n(v) + nU^{n-1}(v') + \frac{n(n-1)}{2!}U^{n-2}u''$$

$$W_2 = y^{n+1}(x) + ny^n(1) + 0.$$

$$w^2 = \underline{\underline{xy^{n+1} + ny^n}}$$

$w^3 = y$.

$u = y$ $u' = y'$ Hence $u^n = y^n$

$v = 1$ $V' = 0$

$$w_3 = U_n(v) + n(U^{n-1}(u')$$

$$= y(1) + 0 = \underline{y}$$

$$w_1 + w_2 + \bar{w}_3 = 0$$

$$xy^{n+2}(x^2) + n\cancel{x}y^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n[n(n-1) + n + 1]$$

$$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$$