$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + 2e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

From the above equation,

Part A

$A = y''$, $A' = y'''$, $A'' = y^{5th}$

part B.

$B = y'(2x+1)$

$u = y'$, $u^* = y^{n+1}$

$v = 2x+1$

$v' = 2$

$v''' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + 2n(y^n)(2) + 0$

$B^n = (2x+1)2y^{n+1} + 2ny^n$

part C

$C = 2y$

$C^n = 2y^n$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1) \Rightarrow$

$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

using leibnitz Theorem

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$, $u^{iv} = 256e^{4x}$

$v = x^3$, $v' = 3x^2$, $v'' = 6x'$, $v''' = 6$, $v^{iv} = 0$

Hence $u^n = 4^n e^{4x}$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2}v^2 + \frac{n(n-1)(n-2)}{8!} u^{n-3} +$$

$$\frac{n(n-1)(n-2)(n-3)u^{n-4}v^2}{4!} +$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4n}(3x^2) + \frac{n(n-1)}{2!}4^{n-2}e^{4n}(6x) + \frac{n(n-1)(n-2)}{3!}$$

$$4^{x-3}e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}(0)^2$$

$$y^n = 4^n x^3 e^{4n} + n3x^2 4^{n-1}e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)$$
$$4^{n-3}e^{4x} + 0$$

$$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4^{4-2} + n(n-1)(n-2))$$

$$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12xn(n-1) + n(n-1)(n-2)) - (y^n)$$

$$y^5 = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + (12x\,5(5-1) + 5(5-1)(5-2))$$

$$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$$

$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

b $$\frac{x^2 d^2y}{dx^2} + \frac{ndy}{dx} + y = 0$$

$x^2 y^x + ny' + y = 0$

$\omega_1 + \omega_2 + \omega_3 = 0$

$\omega_1 = x^2 y''$

$u = y'''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^v$ hence, $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$ $v'' = 2$ $v''' = 0$

$$\omega_1 = u^n(v) + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}(v^{iv}) + \frac{n(n-1)n-2)}{3!}$$

$u^{n-3}v''' + \ldots$

$$\omega_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2}y''(2) + 0.$$

$w_1 = x^2 y^{n+2} + 2xny^{n+1} + y^n n(n-1)$

$w_2 = xy'$

$u = y'$, $u' = y''$; $u'' = y'''$ Have $u'' = y^{x+6}$

$v = x$ $v' = 1$ $v'' = 0$

$w_2 = y^{n+1}(x) + ny^n(1) + 0$

$w_2 = xy^{n+1} + ny^n$

$w_3 = y$

$u = y$ $u' = y'$ Hence $u^n = y^n$

$v = 1$ $v' = 0$

$w_3 = u^n(v) + nu^{n-1}(v')$

$= y(1) + 0 = y$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(n^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n =$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$